Chapter 1

CPE100: Digital Logic Design I

From Zero to One
Background: Digital Logic Design

- How have digital devices changed the world?
- How have digital devices changed your life?
Background

• Digital Devices have revolutionized our world
  • Internet, cell phones, rapid advances in medicine, etc.
• The semiconductor industry has seen tremendous growth
  • $21 billion in 1985 to over $440 billion in 2020
  • Over $520 billion expected in 2021
Chapter 1.1
The Game Plan
The Game Plan

- Purpose of course:
  - Learn the principles of digital design
  - Learn to systematically debug increasingly complex designs
Chapter 1: Topics

- The Art of Managing Complexity
- The Digital Abstraction
- Number Systems
- Addition
- Binary Codes
- Signed Numbers
- Logic Gates
- Logic Levels
- CMOS Transistors
- Power Consumption
Chapter 1.2
The Art of Managing Complexity
The Art of Managing Complexity

- Abstraction
- Discipline
- The Three –y’s
  - Hierarchy
  - Modularity
  - Regularity
Abstraction

- What is abstraction?
  - Hiding details when they are not important

- Electronic computer abstraction
  - Different levels with different building blocks
Discipline

- Intentionally restrict design choices
- Example: Digital discipline
  - Discrete voltages (0 V, 5 V) instead of continuous (0V – 5V)
  - Simpler to design than analog circuits – can build more sophisticated systems
  - Digital systems replacing analog predecessors:
    - i.e., digital cameras, digital television, cell phones, CDs
The Three –y’s

• Hierarchy
  • A system divided into modules and submodules

• Modularity
  • Having well-defined functions and interfaces

• Regularity
  • Encouraging uniformity, so modules can be easily reused
Example: Flintlock Rifle

- Hierarchy

- Three main modules: Lock, stock, and barrel

- Submodules of lock: Hammer, flint, frizzen, etc.
Example Flintlock Rifle

- **Modularity**
  - Function of stock: mount barrel and lock
  - Interface of stock: length and location of mounting pins

- **Regularity**
  - Interchangeable parts
The Art of Managing Complexity

- Abstraction
- Discipline
- The Three –y’s
  - Hierarchy
  - Modularity
  - Regularity
Chapter 1.3
The Digital Abstraction
The Digital Abstraction

• Most physical variables are continuous
  • Voltage on a wire (1.33 V, 9 V, 12.2 V)
  • Frequency of an oscillation (60 Hz, 33.3 Hz, 44.1 kHz)
  • Position of mass (0.25 m, 3.2 m)
• Digital abstraction considers discrete subset of values
  • 0 V, 5 V
  • “0”, “1”
The Analytical Engine

- Designed by Charles Babbage from 1834 – 1871
- Considered to be the first digital computer
- Built from mechanical gears, where each gear represented a discrete value (0-9)
- Babbage died before it was finished
Digital Discipline: Binary Values

• Two discrete values
  • 1 and 0
    • 1 = TRUE = HIGH = ON
    • 0 = FALSE = LOW = OFF
• How to represent 1 and 0
  • Voltage levels, rotating gears, fluid levels, etc.
• Digital circuits use voltage levels to represent 1 and 0
  • Bit = binary digit
    • Represents the status of a digital signal (2 values)
Why Digital Systems?

- Easier to design
- Fast
- Can overcome noise
- Error detection/correction
George Boole, 1815-1864

- Born to working class parents
- Taught himself mathematics and joined the faculty of Queen’s College in Ireland
- Wrote An Investigation of the Laws of Thought (1854)
- Introduced binary variables
- Introduced the three fundamental logic operations: AND, OR, and NOT
Chapter 1.4

Number Systems
Number Systems

- Decimal
  - Base 10
- Binary
  - Base 2
- Hexadecimal
  - Base 16
Decimal Numbers

- Base 10 (our everyday number system)

$$5374_{10} = 5 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$$
Binary Numbers

- Base 2 (computer number system)

1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0

One  Eight One  Four Zero  Two One  One

1's Column
2's Column
4's Column
8's Column

Base 2
Powers of Two

- \(2^0 = \)
- \(2^1 = \)
- \(2^2 = \)
- \(2^3 = \)
- \(2^4 = \)
- \(2^5 = \)
- \(2^6 = \)
- \(2^7 = \)
- \(2^8 = \)
- \(2^9 = \)
- \(2^{10} = \)
- \(2^{11} = \)
- \(2^{12} = \)
- \(2^{13} = \)
- \(2^{14} = \)
- \(2^{15} = \)
Powers of Two

- $2^0 = 1$
- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- $2^7 = 128$

- Handy to memorize up to $2^{10}$

- $2^8 = 256$
- $2^9 = 512$
- $2^{10} = 1024$
- $2^{11} = 2048$
- $2^{12} = 4096$
- $2^{13} = 8192$
- $2^{14} = 16384$
- $2^{15} = 32768$
Bits, Bytes, Nibbles ...

- Bits
- Bytes = 8 bits
- Nibble = 4 bits
- Words = 32 bits
  - Hex digit to represent nibble

10010110

CEBF9AD7

Chapter 1 <27>
Decimal to Binary Conversion

• Two Methods:

• Method 1: Find largest power of 2 that fits, subtract and repeat

• Method 2: Repeatedly divide by 2, remainder goes in next most significant bit
D2B: Method 1

- Find largest power of 2 that fits, subtract, repeat

$53_{10}$
D2B: Method 1

• Find largest power of 2 that fits, subtract, repeat

$53_{10}$  $32 \times 1$
$53 - 32 = 21$  $16 \times 1$
$21 - 16 = 5$  $4 \times 1$
$5 - 4 = 1$  $1 \times 1$

$= 110101_2$
D2B: Method 2

- Repeatedly divide by 2, remainder goes in next most significant bit

\[ 53_{10} = \]
D2B: Method 2

- Repeatedly divide by 2, remainder goes in next most significant bit

\[ 53_{10} = \]
\[ 53/2 = 26 \text{ R1} \]
\[ 26/2 = 13 \text{ R0} \]
\[ 13/2 = 6 \text{ R1} \]
\[ 6/2 = 3 \text{ R0} \]
\[ 3/2 = 1 \text{ R1} \]
\[ 1/2 = 0 \text{ R1} \]

\[ = 110101_2 \]
Number Conversion

- Binary to decimal conversion
  - Convert \( 10011_2 \) to decimal
    \[
    16 \times 1 + 8 \times 0 + 4 \times 0 + 2 \times 1 + 1 \times 1 = 19_{10}
    \]

- Decimal to binary conversion
  - Convert \( 47_{10} \) to binary
    \[
    32 \times 1 + 16 \times 0 + 8 \times 1 + 4 \times 1 + 2 \times 1 + 1 \times 1 = 101111_2
    \]
D2B Example

• Convert $75_{10}$ to binary
D2B Example

• Convert \(75_{10}\) to binary

\[75_{10} = 64 + 8 + 2 + 1 = 1001011_2\]

• Or

\[
\begin{align*}
75/2 &= 37 \quad \text{R1} \\
37/2 &= 18 \quad \text{R1} \\
18/2 &= 9 \quad \text{R0} \\
9/2 &= 4 \quad \text{R1} \\
4/2 &= 2 \quad \text{R0} \\
2/2 &= 1 \quad \text{R0} \\
1/2 &= 0 \quad \text{R1}
\end{align*}
\]
Binary Values and Range

• N-digit decimal number
  • How many values?
  • Range?

• Example:
  3-digit decimal number
  • Possible values
  • Range
Binary Values and Range

- **N-digit decimal number**
  - **How many values?**
    - \(10^N\)
  - **Range?**
    - \([0, 10^N - 1]\)

- **Example:**
  3-digit decimal number
  - **Possible values**
    - \(10^3 = 1000\)
  - **Range**
    - \([0, 999]\)
Binary Values and Range

- N-bit binary number
  - How many values?
    - Range?

- Example:
  3-bit binary number
  - Possible values
    - Range
Binary Values and Range

- N-bit binary number
  - How many values?
    - $2^N$
  - Range?
    - $[0, 2^N - 1]$

- Example:
  3-bit binary number
  - Possible values
    - $2^3 = 8$
  - Range
    - $[0, 7] = [000_2, 111_2]$
Binary Values and Range

- **N-digit decimal number**
  - How many values?
    - \(10^N\)
  - Range?
    - \([0, 10^N - 1]\)

- **Example:**
  3-digit decimal number
  - Possible values
    - \(10^3 = 1000\)
  - Range
    - \([0, 999]\)

- **N-bit binary number**
  - How many values?
    - \(2^N\)
  - Range?
    - \([0, 2^N - 1]\)

- **Example:**
  3-bit binary number
  - Possible values
    - \(2^3 = 8\)
  - Range
    - \([0, 7] = [000_2, 111_2]\)
Hexadecimal Numbers

- Base 16 number system
- Shorthand for binary
  - Four binary digits (4-bit binary number) is a single hex digit
## Hexadecimal Numbers

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<th>Hex Digit</th>
<th>Decimal Equivalent</th>
<th>Binary Equivalent</th>
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<td>Hex Digit</td>
<td>Decimal Equivalent</td>
<td>Binary Equivalent</td>
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<tr>
<td>F</td>
<td>15</td>
<td>1111</td>
</tr>
</tbody>
</table>
Hexadecimal to Binary Conversion

• Hexadecimal to binary conversion:
  • Convert 4AF₁₆ (also written 0x4AF) to binary

• Hexadecimal to decimal conversion:
  • Convert 0x4AF to decimal
Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
  - Convert \(4AF_{16}\) (also written 0x4AF) to binary
    - \(0x4AF = 0100\ 1010\ 1111_2\)

- Hexadecimal to decimal conversion:
  - Convert 0x4AF to decimal
    - \(4 \times 16^2 + 10 \times 16^1 + 15 \times 16^0 = 1199_{10}\)
Number Systems

- **Popular**
  - Decimal Base 10
  - Binary Base 2
  - Hexadecimal Base 16

- **Others**
  - Octal Base 8
  - Any other base
Octal Numbers

- Same as hex with one less binary digit

<table>
<thead>
<tr>
<th>Octal Digit</th>
<th>Decimal Equivalent</th>
<th>Binary Equivalent</th>
</tr>
</thead>
<tbody>
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<td>110</td>
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<tr>
<td>7</td>
<td>7</td>
<td>111</td>
</tr>
</tbody>
</table>
Number Systems

- In general, an N-digit number \( \{a_{N-1}a_{N-2} \ldots a_1a_0\} \) of base \( R \) in decimal equals
  \[ a_{N-1}R^{N-1} + a_{N-2}R^{N-2} + \ldots + a_1R^1 + a_0R^0 \]
- Example: 4-digit \( \{5173\} \) of base 8 (octal)
Number Systems

• In general, an N-digit number \( \{a_{N-1}a_{N-2} \ldots a_1a_0\} \) of base \( R \) in decimal equals
  \[
a_{N-1}R^{N-1} + a_{N-2}R^{N-2} + \cdots + a_1R^1 + a_0R^0
\]

• Example: 4-digit \( \{5173\} \) of base 8 (octal)
  \[
  5 \times 8^3 + 1 \times 8^2 + 7 \times 8^1 + 3 \times 8^0 = 2683_{10}
  \]
Decimal to Octal Conversion

• Remember two methods for D2B conversion
  • 1: remove largest multiple; 2: repeated divide
• Convert $29_{10}$ to octal
Decimal to Octal Conversion

- Remember two methods for D2B conversion
  - 1: remove largest multiple; 2: repeated divide
- Convert $29_{10}$ to octal
- Method 2

\[
\begin{align*}
29 \div 8 &= 3 \text{ R}5 \text{ lsb} \\
3 \div 8 &= 0 \text{ R}3 \text{ msb}
\end{align*}
\]

$29_{10} = 35_8$
Decimal to Octal Conversion

- Remember two methods for D2B conversion
  - 1: remove largest multiple; 2: repeated divide
- Convert $29_{10}$ to octal
- Method 1

\[
29_{10} = 24 + 5 = 3 \times 8^1 + 5 \times 8^0 = 35_8
\]

- Or (better scalability)

\[
29_{10} = 16 + 8 + 4 + 1 = 11101_2 = 35_8
\]
Octal to Decimal Conversion

- Convert $163_8$ to decimal
Octal to Decimal Conversion

• Convert $163_8$ to decimal

  • $163_8 = 1 \times 8^2 + 6 \times 8^1 + 3$
  • $163_8 = 64 + 48 + 3$
  • $163_8 = 115_{10}$
Recap: Binary and Hex Numbers

- Example 1: Convert $83_{10}$ to hex
- Example 2: Convert $01101011_2$ to hex and decimal
- Example 3: Convert $0xCA3$ to binary and decimal
Recap: Binary and Hex Numbers

• Example 1: Convert $83_{10}$ to hex
  • $83_{10} = 64 + 16 + 2 + 1 = 1010011_2$
  • $1010011_2 = 1010011_2 = 53_{16}$

• Example 2: Convert $01101011_2$ to hex and decimal
  • $01101011_2 = 0110 1011_2 = 6B_{16}$
  • $0x6B = 6 \times 16^1 + 11 \times 16^0 = 96 + 11 = 107$

• Example 3: Convert $0xCA3$ to binary and decimal
  • $0xCA3 = 1100 1010 0011_2$
  • $0xCA3 = 12 \times 16^2 + 10 \times 16^1 + 3 \times 16^0 = 3235_{10}$
Large Powers of Two

- $2^{10} = 1 \text{kilo} \approx 1000 \ (1024)$
- $2^{20} = 1 \text{mega} \approx 1 \text{million} \ (1,048,576)$
- $2^{30} = 1 \text{giga} \approx 1 \text{billion} \ (1,073,741,824)$
- $2^{40} = 1 \text{tera} \approx 1 \text{trillion} \ (1,099,511,627,776)$
Large Powers of Two: Abbreviations

- $2^{10} = 1 \text{kilo} \approx 1000 \ (1024)$
  
  for example: 1 kB = 1024 Bytes
  1 kb = 1024 bits

- $2^{20} = 1 \text{mega} \approx 1 \text{million} \ (1,048,576)$
  
  for example: 1 MiB, 1 Mib (1 megabit)

- $2^{30} = 1 \text{giga} \approx 1 \text{billion} \ (1,073,741,824)$
  
  for example: 1 GiB, 1 Gib
Estimating Powers of Two

• What is the value of $2^{24}$?

• How many values can a 32-bit variable represent?
Estimating Powers of Two

• What is the value of $2^{24}$?
  • $2^4 \times 2^{20} \approx 16$ million

• How many values can a 32-bit variable represent?
  • $2^2 \times 2^{30} \approx 4$ billion
Binary Codes

Another way of representing decimal numbers in binary

Example binary codes:

• Weighted codes
  • Binary Coded Decimal (BCD) (8-4-2-1 code)
  • 6-3-1-1 code
  • 8-4-2-1 code (simple binary)

• Gray codes
• Excess-3 code
• 2-out-of-5 code
### Table 1-3: ASCII Code

<table>
<thead>
<tr>
<th>Character</th>
<th>ASCII Code</th>
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<td>1 0 0 0 0 0 1</td>
<td>v</td>
<td>1 1 0 0 0 0 1</td>
</tr>
<tr>
<td>8</td>
<td>0 1 0 0 0 0 1</td>
<td>W</td>
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<td>w</td>
<td>1 1 0 0 0 0 1</td>
</tr>
<tr>
<td>9</td>
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<td>X</td>
<td>1 0 0 0 0 0 1</td>
<td>x</td>
<td>1 1 0 0 0 0 1</td>
</tr>
<tr>
<td>:</td>
<td>0 1 0 0 0 0 1</td>
<td>Y</td>
<td>1 0 0 0 0 0 1</td>
<td>y</td>
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<tr>
<td>;</td>
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<tr>
<td>&lt;</td>
<td>0 1 0 0 0 0 1</td>
<td>[</td>
<td>1 0 0 0 0 0 1</td>
<td></td>
<td>1 1 0 0 0 0 1</td>
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<tr>
<td>=</td>
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<td>\</td>
<td>1 0 0 0 0 0 1</td>
<td></td>
<td>1 1 0 0 0 0 1</td>
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<tr>
<td>&gt;</td>
<td>0 1 0 0 0 0 1</td>
<td>]</td>
<td>1 0 0 0 0 0 1</td>
<td></td>
<td>1 1 0 0 0 0 1</td>
</tr>
<tr>
<td>?</td>
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<td>^</td>
<td>1 0 0 0 0 0 1</td>
<td></td>
<td>1 1 0 0 0 0 1</td>
</tr>
<tr>
<td>@</td>
<td>0 1 0 0 0 0 1</td>
<td>_</td>
<td>1 0 0 0 0 0 1</td>
<td>delete</td>
<td>1 1 1 1 1 1 1</td>
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</table>
Binary Codes

<table>
<thead>
<tr>
<th>Decimal #</th>
<th>8-4-2-1 (BCD)</th>
<th>6-3-1-1</th>
<th>Excess-3</th>
<th>2-out-of-5</th>
<th>Gray</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0000</td>
<td>0000</td>
<td>0011</td>
<td>00011</td>
<td>0000</td>
</tr>
<tr>
<td>1</td>
<td>0001</td>
<td>0001</td>
<td>0100</td>
<td>00101</td>
<td>0001</td>
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<td>0101</td>
<td>00110</td>
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<tr>
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<td>0011</td>
<td>0100</td>
<td>0110</td>
<td>01001</td>
<td>0010</td>
</tr>
<tr>
<td>4</td>
<td>0100</td>
<td>0101</td>
<td>0111</td>
<td>01010</td>
<td>0110</td>
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<tr>
<td>5</td>
<td>0101</td>
<td>0111</td>
<td>1000</td>
<td>01100</td>
<td>1110</td>
</tr>
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<td>6</td>
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<td>7</td>
<td>0111</td>
<td>1001</td>
<td>1010</td>
<td>10010</td>
<td>1011</td>
</tr>
<tr>
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<td>1000</td>
<td>1011</td>
<td>1011</td>
<td>10100</td>
<td>1001</td>
</tr>
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<td>9</td>
<td>1001</td>
<td>1100</td>
<td>1100</td>
<td>11000</td>
<td>1000</td>
</tr>
</tbody>
</table>

Each code combination represents a single decimal digit.
Gray Codes

- Next number differs in only one bit position
  - Example: 000, 001, 011, 010, 110, 111, 101, 100

- Example use: Analog-to-Digital (A/D) converters. Changing 2 bits at a time (i.e., 011 → 100) could cause large inaccuracies.

- Will use in K-maps

<table>
<thead>
<tr>
<th>Decimal #</th>
<th>Gray</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0000</td>
</tr>
<tr>
<td>1</td>
<td>0001</td>
</tr>
<tr>
<td>2</td>
<td>0011</td>
</tr>
<tr>
<td>3</td>
<td>0010</td>
</tr>
<tr>
<td>4</td>
<td>0110</td>
</tr>
<tr>
<td>5</td>
<td>1110</td>
</tr>
<tr>
<td>6</td>
<td>1010</td>
</tr>
<tr>
<td>7</td>
<td>1011</td>
</tr>
<tr>
<td>8</td>
<td>1001</td>
</tr>
<tr>
<td>9</td>
<td>1000</td>
</tr>
</tbody>
</table>
Chapter 1.4.5
Addition
Addition

- Decimal

\[
\begin{align*}
3734 & \\
+ 5168 & \\
\hline
1011 & \\
\end{align*}
\]

- Binary

\[
\begin{align*}
1011 & \\
+ 0011 & \\
\hline
0011 & \\
\end{align*}
\]
Addition

- Decimal
  
  \[
  \begin{array}{c}
  3734 \\
  + 5168 \\
  \hline
  8902
  \end{array}
  \]

  \[\text{carries} \quad 11\]

- Binary

  \[
  \begin{array}{c}
  1011 \\
  + 0011 \\
  \hline
  10010
  \end{array}
  \]
Addition

- Decimal

\[
\begin{array}{c}
3734 \\
+ \quad 5168 \\
\hline
8902
\end{array}
\]

11 \leftarrow \text{carries}

- Binary

\[
\begin{array}{c}
1011 \\
+ \quad 0011 \\
\hline
1110
\end{array}
\]

11 \leftarrow \text{carries}
Binary Addition Examples

• Add the following 4-bit binary numbers

  1001
  + 0101
  ________
  1011

• Add the following 4-bit binary numbers

  1011
  + 0110
  ________
Binary Addition Examples

• Add the following 4-bit binary numbers

\[
\begin{array}{cccc}
1 & 0 & 0 & 1 \\
+ & 0 & 1 & 0 & 1 \\
\hline
& & 1 & 1 & 1 & 0
\end{array}
\]

• Add the following 4-bit binary numbers

\[
\begin{array}{cccc}
1 & 0 & 1 & 1 \\
+ & 0 & 1 & 1 & 0 \\
\hline
& & 1 & 1 & 0 & 0
\end{array}
\]
Binary Addition Examples

• Add the following 4-bit binary numbers

\[
\begin{align*}
1001 \\
+ 0101 \\
\hline
1110
\end{align*}
\]

Overflow!

• Add the following 4-bit binary numbers

\[
\begin{align*}
111 \\
+ 1011 \\
+ 0110 \\
\hline
10001
\end{align*}
\]

Overflow!
Overflow

- Digital systems operate on a **fixed number of bits**
- Overflow: when result is too big to fit in the available number of bits
- See previous example of 11 + 6
Signed Binary Numbers

- Sign/Magnitude Numbers
- Two’s Complement Numbers
Sign/Magnitude

• 1 sign bit, $N$-1 magnitude bits
• Sign bit is the most significant (left-most) bit
  – Positive number: sign bit = 0
  – Negative number: sign bit = 1

• Example, 4-bit sign/magnitude representations of $\pm 6$:
  • $+6 = \cdots$
  • $-6 = \cdots$

• Range of an $N$-bit sign/magnitude number:

\[
A = (-1)^{a_{n-1}} \sum_{i=0}^{n-2} a_i 2^i
\]
Sign/Magnitude

- 1 sign bit, $N$-1 magnitude bits
- Sign bit is the most significant (left-most) bit
  - Positive number: sign bit = 0
  - Negative number: sign bit = 1

Example, 4-bit sign/magnitude representations of ±6:
- +6 = 0110
- -6 = 1110

Range of an $N$-bit sign/magnitude number:
- $[-(2^{N-1}-1), 2^{N-1}-1]$
Sign/Magnitude Numbers

• Problems:
  • Addition doesn’t work, for example -6 + 6:
    \[ 1110 + 0110 = 1100 \]
  • Two representations of 0 (± 0):
    • (+0) =
    • (−0) =
Sign/Magnitude Numbers

• Problems:
  • Addition doesn’t work, for example -6 + 6:
    \[
    \begin{align*}
    1110 \\
    + 0110 \\
    \hline
    10100 \text{ (wrong!)}
    \end{align*}
    \]
  • Two representations of 0 (± 0):
    • (+0) = 0000
    • (−0) = 1000
Two’s Complement Numbers

• Don’t have same problems as sign/magnitude numbers:
  • Addition works
  • Single representation for 0

• Range of representable numbers not symmetric
  • One extra negative number
Two’s Complement Numbers

- msb has value of \(-2^{n-1}\)
  \[
  A = a_{n-1}(-2^{n-1}) + \sum_{i=0}^{n-2} a_i 2^i
  \]

- The most significant bit still indicates the sign (1 = negative, 0 = positive)

- Range of an \(N\)-bit two’s comp number?

- Most positive 4-bit number?
- Most negative 4-bit number?
Two’s Complement Numbers

- msb has value of $-2^{N-1}$

$$A = a_{n-1}(-2^{n-1}) + \sum_{i=0}^{n-2} a_i 2^i$$

- The most significant bit still indicates the sign (1 = negative, 0 = positive)
- Range of an $N$-bit two’s comp number?
  - $[-(2^{N-1}), 2^{N-1} - 1]$
- Most positive 4-bit number? 0111
- Most negative 4-bit number? 1000
"Taking the Two’s Complement"

- **Flips the sign** of a two’s complement number
- **Method:**
  1. Invert the bits
  2. Add 1
- **Example:** Flip the sign of $3_{10} = 0011_2$
“Taking the Two’s Complement”

- Flips the sign of a two’s complement number
- Method:
  1. Invert the bits
  2. Add 1
- Example: Flip the sign of $3_{10} = 0011_2$
  1. $1100$
  2. $+ 1$
  $$1101 = -3_{10}$$
Two’s Complement Examples

- Take the two’s complement of $6_{10} = 0110_2$

- What is the decimal value of the two’s complement number $1001_2$?
Two’s Complement Examples

- Take the two’s complement of $6_{10} = 0110_2$
  1. 1001
  2. $+ 1$
  \[ 1010_2 = -6_{10} \]

- What is the decimal value of the two’s complement number $1001_2$?
  1. 0110
  2. $+ 1$
  \[ 0111_2 = 7_{10}, \text{ so } 1001_2 = -7_{10} \]
Two’s Complement Addition

- Add 6 + (-6) using two’s complement numbers

\[
\begin{array}{c}
0110 \\
+ 1010
\end{array}
\]

- Add -2 + 3 using two’s complement numbers

\[
\begin{array}{c}
1110 \\
+ 0011
\end{array}
\]
Two’s Complement Addition

- Add 6 + (-6) using two’s complement numbers

\[
\begin{align*}
&111 \\
&0110 \\
+ &1010 \\
\hline
&10000
\end{align*}
\]

- Add -2 + 3 using two’s complement numbers

\[
\begin{align*}
&1110 \\
+ &0011 \\
\hline
&10011
\end{align*}
\]
Two’s Complement Addition

• Add 6 + (-6) using two’s complement numbers
  
  \[
  \begin{array}{c}
  111 \\
  0110 \\
  + 1010 \\
  \hline
  10000
  \end{array}
  \]

• Add -2 + 3 using two’s complement numbers
  
  \[
  \begin{array}{c}
  111 \\
  1110 \\
  + 0011 \\
  \hline
  10001
  \end{array}
  \]
Increasing Bit Width

• Extend number from N to M bits (M > N):
  • Sign-extension
  • Zero-extension
Sign-Extension

- Sign bit copied to msb’s
- Number value is same

- Example 1
  - 4-bit representation of 3 = 0011
  - 8-bit sign-extended value:

- Example 2
  - 4-bit representation of -7 = 1001
  - 8-bit sign-extended value:
Sign-Extension

- Sign bit copied to msb’s
- Number value is same

- Example 1
  - 4-bit representation of 3 = 0011
  - 8-bit sign-extended value: 00000011

- Example 2
  - 4-bit representation of -7 = 1001
  - 8-bit sign-extended value: 11111001
Zero-Extension

• Zeros copied to msb’s
• Value changes for negative numbers

• Example 1
  • 4-bit value = $0011_2$
  • 8-bit zero-extended value:

• Example 2
  • 4-bit value = $1001$
  • 8-bit zero-extended value:
Zero-Extension

- Zeros copied to msb’s
- Value changes for negative numbers

- Example 1
  - 4-bit value = \(0011\)
  - 8-bit zero-extended value: \(00000011\)

- Example 2
  - 4-bit value = \(1001\)
  - 8-bit zero-extended value: \(00001001\)
Zero-Extension

- Zeros copied to msb’s
- Value changes for negative numbers

- Example 1
  - 4-bit value = 0011₂ = 3₁₀
  - 8-bit zero-extended value: 00000011 = 3₁₀

- Example 2
  - 4-bit value = 1001 = -7₁₀
  - 8-bit zero-extended value: 00001001 = 9₁₀
## Number System Comparison

<table>
<thead>
<tr>
<th>Number System</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsigned</td>
<td>[0, (2^N-1)]</td>
</tr>
<tr>
<td>Sign/Magnitude</td>
<td>([-2^{N-1}-1, 2^{N-1}-1])</td>
</tr>
<tr>
<td>Two’s Complement</td>
<td>([-2^{N-1}, 2^{N-1}-1])</td>
</tr>
</tbody>
</table>

For example, 4-bit representation:

<table>
<thead>
<tr>
<th>Unsigned</th>
<th>0000 0001 0010 0011 0100 0101 0110 0111 1000 1001 1010 1011 1100 1101 1110 1111</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two's Complement</td>
<td>1000 1001 1010 1011 1100 1101 1110 1111 0000 0001 0010 0011 0100 0101 0110 0111</td>
</tr>
<tr>
<td>Sign/Magnitude</td>
<td>1111 1110 1101 1100 1011 1010 1001 0000 1000 0001 0010 0011 0100 0101 0110 0111</td>
</tr>
</tbody>
</table>
Chapter 1.5

Logic Gates
Logic Gates

- Perform logic functions:
  - inversion (NOT), AND, OR, NAND, NOR, etc.
- Single-input:
  - NOT gate, buffer
- Two-input:
  - AND, OR, XOR, NAND, NOR, XNOR
- Multiple-input
Single-Input Logic Gates

**NOT**

\[
Y = \overline{A}
\]

<table>
<thead>
<tr>
<th>A</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

**BUF**

\[
Y = A
\]

<table>
<thead>
<tr>
<th>A</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Single-Input Logic Gates

• Bubble on wire indicates inversion

\[ Y = \overline{A} \]

\[
\begin{array}{c|c}
A & Y \\
0 & 1 \\
1 & 0 \\
\end{array}
\]

• Note: bar over variable indicates complement (invert value)

\[ Y = A \]

\[
\begin{array}{c|c}
A & Y \\
0 & 0 \\
1 & 1 \\
\end{array}
\]
Two-Input Logic Gates

AND

\[ Y = AB \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

OR

\[ Y = A + B \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
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<td>0</td>
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</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Two-Input Logic Gates

**AND**

\[ Y = AB \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**OR**

\[ Y = A + B \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
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<td>0</td>
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</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
More Two-Input Logic Gates

**XOR**

\[ Y = A \oplus B \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
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<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

**NAND**

\[ Y = \overline{AB} \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

**NOR**

\[ Y = \overline{A + B} \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
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<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**XNOR**

\[ Y = A \oplus B \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<tr>
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</tr>
<tr>
<td>1</td>
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</tr>
</tbody>
</table>
More Two-Input Logic Gates

<table>
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<th>A</th>
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<tbody>
<tr>
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**XOR**

$$Y = A \oplus B$$

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<th>Y</th>
</tr>
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<tr>
<td>0</td>
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<td>0</td>
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</tbody>
</table>

**NAND**

$$Y = \overline{AB}$$

<table>
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<tr>
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</tr>
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</tbody>
</table>

**NOR**

$$Y = \overline{A + B}$$

<table>
<thead>
<tr>
<th>A</th>
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<th>Y</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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**XNOR**

$$Y = \overline{A \oplus B}$$

<table>
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</table>
Multiple-Input Logic Gates

**NOR3**

\[
    Y = \overline{A+B+C}
\]

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<th>C</th>
<th>Y</th>
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**AND3**

\[
    Y = ABC
\]

<table>
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<th>C</th>
<th>Y</th>
</tr>
</thead>
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</table>
Multiple-Input Logic Gates

- Multi-input XOR = Odd parity (#on inputs odd $\Rightarrow 1$)

**NOR3**

$$Y = \overline{A + B + C}$$

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
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<th>Y</th>
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**AND3**

$$Y = ABC$$

<table>
<thead>
<tr>
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<th>B</th>
<th>C</th>
<th>Y</th>
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<tr>
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<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Chapter 1.6

Beneath the Digital Abstraction
Logic Levels

• Discrete voltages represent 1 and 0
• For example:
  • 0 = ground (GND) or 0 volts
  • 1 = $V_{DD}$ or 5 volts
• What about 4.99 volts? Is that a 0 or a 1?
• What about 3.2 volts?
Logic Levels

• Must have *range* of voltages for 1 and 0

• Different ranges for inputs and outputs to allow for *noise*
What is Noise?

- Anything that degrades the signal
  - E.g., resistance, power supply noise, coupling to neighboring wires, etc.

- Example: a gate (driver) outputs 5 V but, because of resistance in a long wire, receiver gets 4.5 V
The Static Discipline

- With logically valid inputs, every circuit element must produce logically valid outputs.

- Use limited ranges of voltages to represent discrete values.
Real Logic Levels

- Want driver to output “clean” high/low and receiver to handle noisy high/low
Real Logic Levels

- Want driver to output “clean” high/low and receiver to handle noisy high/low.
Real Logic Levels

Driver Receiver

Output Characteristics

Input Characteristics

Logic High

Logic Low

Input Range

Output Range

Logic High

Logic Low

Input Characteristics

Output Characteristics

Forbidden Zone

\[ NM_H = V_{OH} - V_{IH} \]

\[ NM_L = V_{IL} - V_{OL} \]


V_{DD} Scaling

- In 1970’s and 1980’s, $V_{DD} = 5$ V
- $V_{DD}$ has dropped
  - 3.3 V, 2.5 V, 1.8 V, 1.5 V, 1.2 V, 1.0 V, ...

- Avoid frying tiny transistors
- Save power

- Be careful connecting chips with different supply voltages
  - Easy to fry if not careful
# Logic Family Examples

<table>
<thead>
<tr>
<th>Logic Family</th>
<th>$V_{DD}$</th>
<th>$V_{IL}$</th>
<th>$V_{IH}$</th>
<th>$V_{OL}$</th>
<th>$V_{OH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTL</td>
<td>5 (4.75 - 5.25)</td>
<td>0.8</td>
<td>2.0</td>
<td>0.4</td>
<td>2.4</td>
</tr>
<tr>
<td>CMOS</td>
<td>5 (4.5 - 6)</td>
<td>1.35</td>
<td>3.15</td>
<td>0.33</td>
<td>3.84</td>
</tr>
<tr>
<td>LVTTL</td>
<td>3.3 (3 - 3.6)</td>
<td>0.8</td>
<td>2.0</td>
<td>0.4</td>
<td>2.4</td>
</tr>
<tr>
<td>LVCMOS</td>
<td>3.3 (3 - 3.6)</td>
<td>0.9</td>
<td>1.8</td>
<td>0.36</td>
<td>2.7</td>
</tr>
</tbody>
</table>
Chapter 1.7

CMOS Transistors
Transistors

• Logic gates built from transistors
• Simple model: 3-ported voltage-controlled switch
  • 2 ports connected depending on voltage of 3rd
  • d and s are connected (ON) when g is 1
Robert Noyce, 1927-1990

- Nicknamed “Mayor of Silicon Valley”
- Cofounded Fairchild Semiconductor in 1957
- Cofounded Intel in 1968
- Co-invented the integrated circuit
Transistors built from silicon, a semiconductor
Pure silicon is a poor conductor (no free charges)
Doped silicon is a good conductor (free charges)
- n-type (free negative charges, electrons)
- p-type (free positive charges, holes)
MOS Transistors

- Metal oxide silicon (MOS) transistors:
  - Polysilicon (used to be metal) gate
  - Oxide (silicon dioxide) insulator
  - Doped silicon
nMOS Transistors

- Gate = 0
- OFF (no connection between source and drain)

- Gate = 1
- ON (channel between source and drain)

Diode connection from p to n doped area → current cannot travel from n→p
pMOS Transistors

- pMOS transistor is opposite of nMOS
  - ON when Gate = 0
  - OFF when Gate = 1

Note bubble on gate to indicate on when low
Transistor Function

• Voltage controlled switch

\[
\begin{align*}
\text{nMOS} & \quad g = 0 \quad \text{OFF} \\
& \quad g = 1 \quad \text{ON} \\
\text{pMOS} & \quad g = 0 \quad \text{OFF} \\
& \quad g = 1 \quad \text{ON}
\end{align*}
\]
Transistor Composition

- **nMOS**: pass good 0’s
  - Connect source to GND
  - “Pull down” transistor

- **pMOS**: pass good 1’s
  - Connect source to VDD
  - “Pull up” transistor

- Build logic gates from composition
  - CMOS = complementary MOS
CMOS Gate Structure

- Pull-up pMOS network connects to $V_{DD}$
- Pull-down nMOS network connects to $GND$
- Use series and parallel connections to implement gate logic
CMOS Gates: NOT Gate

\[ Y = \overline{A} \]

<table>
<thead>
<tr>
<th>A</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ V_{DD} \]

\[ A \quad P1 \quad N1 \quad Y \]

<table>
<thead>
<tr>
<th>A</th>
<th>P1</th>
<th>N1</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
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</tbody>
</table>
CMOS Gates: NOT Gate

\[ Y = \overline{A} \]

<table>
<thead>
<tr>
<th>A</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>1</td>
<td>0</td>
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</table>

<table>
<thead>
<tr>
<th>A</th>
<th>P1</th>
<th>N1</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>ON</td>
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<td>1</td>
</tr>
<tr>
<td>1</td>
<td>OFF</td>
<td>ON</td>
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</table>
CMOS Gates: NAND Gate

NAND

\[ Y = \overline{AB} \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

A
B
Y
N2
N1
P2 P1

\[
\begin{array}{ccc}
A & B & P1 & P2 & N1 & N2 & Y \\
0 & 0 & & & & & \\
0 & 1 & & & & & \\
1 & 0 & & & & & \\
1 & 1 & & & & & \\
\end{array}
\]
CMOS Gates: NAND Gate

NAND

\[ Y = \overline{AB} \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
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</thead>
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<table>
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<th>N1</th>
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</tr>
</tbody>
</table>
CMOS Gates: NOR Gate

- How can you build three input \((A, B, C)\) NOR gate?
CMOS Gates: NOR Gate

• How can you build three input \((A, B, C)\) NOR gate?

Only high output when all three pMOS in series are “on” and create a path from output to \(V_{DD}\)
CMOS Gates: AND Gate

• How can you build 2 input AND gate?
CMOS Gates: AND Gate

• How can you build 2 input AND gate?

A

B

Y
CMOS Gates: AND Gate

- How can you build 2 input AND gate?

![AND Gate Diagram]

Note: AND requires 2 more gates than NAND. Inverted logic is more efficient implementation.
Transmission Gates

- nMOS pass 1’s poorly, pMOS pass 0’s poorly
- Transmission gate is for passing signal
  - Pass both 0 and 1 well
- When EN = 1, the switch is ON:
  - $\overline{EN} = 0$ and A is connected to B
- When EN = 0, the switch is OFF:
  - A is not connected to B
Psuedo-nMOS

• Replace pull-up network with weak pMOS transistor that is always on
  • pMOS gate tied to ground
• pMOS transistor: pulls output HIGH only when nMOS network not pulling it LOW
Psuedo-nMOS Example: NOR4

- How many transistors needed?
Psuedo-nMOS Example: NOR4

- How many transistors needed?
  - Only 5 since a single pMOS is used
Chapter 1.8

Power Consumption
Gordon Moore, 1929-

- Cofounded Intel in 1968 with Robert Noyce.
- Moore’s Law: number of transistors on a computer chip doubles every year (observed in 1965)
  - Since 1975, transistor counts have doubled every two years.
Moore's Law

• Transistor count doubles every 2 years

Microprocessor Transistor Counts 1971-2011 & Moore's Law
Moore’s Law Trends

- “If the automobile had followed the same development cycle as the computer, a Rolls-Royce would today cost $100, get one million miles to the gallon, and explode once a year . . .”
  – Robert Cringley
Power Consumption

• Power = Energy consumed per unit time

• Two types of power
  • Dynamic power consumption
  • Static power consumption
Dynamic Power Consumption

- Power to charge transistor gate capacitances
  - Energy required to charge a capacitance, \( C \), to \( V_{DD} \) is \( CV_{DD}^2 \)
  - Circuit running at frequency \( f \): transistors switch (from 1 to 0 or vice versa) at that frequency
  - Capacitor is charged \( f/2 \) times per second (discharging from 1 to 0 is free)

- Dynamic power consumption
  \[
P_{\text{dynamic}} = \frac{1}{2} CV_{DD}^2 f
\]
Static Power Consumption

• Power consumed when no gates are switching

• Caused by the quiescent supply current, $I_{DD}$ (also called the leakage current)

• Static power consumption

$$P_{static} = I_{DD}V_{DD}$$
Power Consumption Example

- Estimate the power consumption of a wireless handheld computer
  - $V_{DD} = 1.2 \text{ V}$
  - $C = 20 \text{ nF}$
  - $f = 1 \text{ GHz}$
  - $I_{DD} = 20 \text{ mA}$

- Total power is sum of dynamic and static
Power Consumption Example

- Estimate the power consumption of a wireless handheld computer
  - \( V_{DD} = 1.2 \) V
  - \( C = 20 \) nF
  - \( f = 1 \) GHz
  - \( I_{DD} = 20 \) mA

- Total power is sum of dynamic and static

\[
P = \frac{1}{2} CV_{DD}^2 f + I_{DD} V_{DD}
\]

\[
= \frac{1}{2} (20 \text{ nF})(1.2)^2 (1 \text{ GHz})
\]

\[
+ (20 \text{ mA})(1.2)
\]

\[
= (14.4 + 0.024) \text{W}
\]

\[
= 14.4 \text{ W}
\]
Extras
Binary Codes

Another way of representing decimal numbers

Example binary codes:

• Weighted codes
  • Binary Coded Decimal (BCD) (8-4-2-1 code)
  • 6-3-1-1 code
  • Simple binary

• Gray codes

• Excess-3 code

• 2-out-of-5 code
### Binary Codes

<table>
<thead>
<tr>
<th>Decimal #</th>
<th>8-4-2-1 (BCD)</th>
<th>6-3-1-1</th>
<th>Excess-3</th>
<th>2-out-of-5</th>
<th>Gray</th>
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<td>1100</td>
<td>1100</td>
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</table>

Each code combination represents a **single decimal digit**.
# ASCII-Code

<table>
<thead>
<tr>
<th>Character</th>
<th>ASCII Code</th>
<th>Character</th>
<th>ASCII Code</th>
<th>Character</th>
<th>ASCII Code</th>
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<td>@</td>
<td>1 0 0 0 0 0 0</td>
<td>'</td>
<td>1 1 0 0 0 0 0</td>
</tr>
<tr>
<td>!</td>
<td>0 1 0 0 0 0 1</td>
<td>A</td>
<td>1 0 0 0 0 0 1</td>
<td>a</td>
<td>1 1 0 0 0 0 1</td>
</tr>
<tr>
<td>&quot;</td>
<td>0 1 0 0 0 0 1</td>
<td>B</td>
<td>1 0 0 0 0 1 0</td>
<td>b</td>
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<tr>
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<td>c</td>
<td>1 1 0 0 0 1 1</td>
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</table>
Weighted Codes

- Weighted codes: each bit position has a given weight
  - Binary Coded Decimal (BCD) (8-4-2-1 code)
    - Example: $726_{10} = 0111\ 0010\ 0110_{\text{BCD}}$
  - 6-3-1-1 code
    - Example: 1001 (6-3-1-1 code) = $1\times6 + 0\times3 + 0\times1 + 1\times1$
    - Example: $726_{10} = 1001\ 0011\ 1000_{6311}$

- BCD numbers are used to represent fractional numbers exactly (vs. floating point numbers – which can’t - see Chapter 5)
Weighted Codes

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<thead>
<tr>
<th>Decimal #</th>
<th>8-4-2-1 (BCD)</th>
<th>6-3-1-1</th>
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</table>

- **BCD Example:**
  \[ 726_{10} = 0111 \ 0010 \ 0110_{BCD} \]

- **6-3-1-1 code Example:**
  \[ 726_{10} = 1001 \ 0011 \ 1000_{6311} \]
Excess-3 Code

- Add 3 to number, then represent in binary
  - Example: $5_{10} = 5+3 = 8 = 1000_2$
- Also called a biased number
- Excess-3 codes (also called XS-3) were used in the 1970’s to ease arithmetic

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<td>8</td>
<td>1011</td>
</tr>
<tr>
<td>9</td>
<td>1100</td>
</tr>
</tbody>
</table>

- Excess-3 Example: $726_{10} = \text{1010 0101 1001}_{xs3}$
### 2-out-of-5 Code

- 2 out of the 5 bits are 1
- Used for error detection:
  - If more or less than 2 of 5 bits are 1, error

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Gray Codes

- Next number differs in only one bit position
  - **Example:** 000, 001, 011, 010, 110, 111, 101, 100

- **Example use:** Analog-to-Digital (A/D) converters. Changing 2 bits at a time (i.e., 011 → 100) could cause large inaccuracies.

<table>
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