Chapter 1

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CPE100: Digital Logic Design I

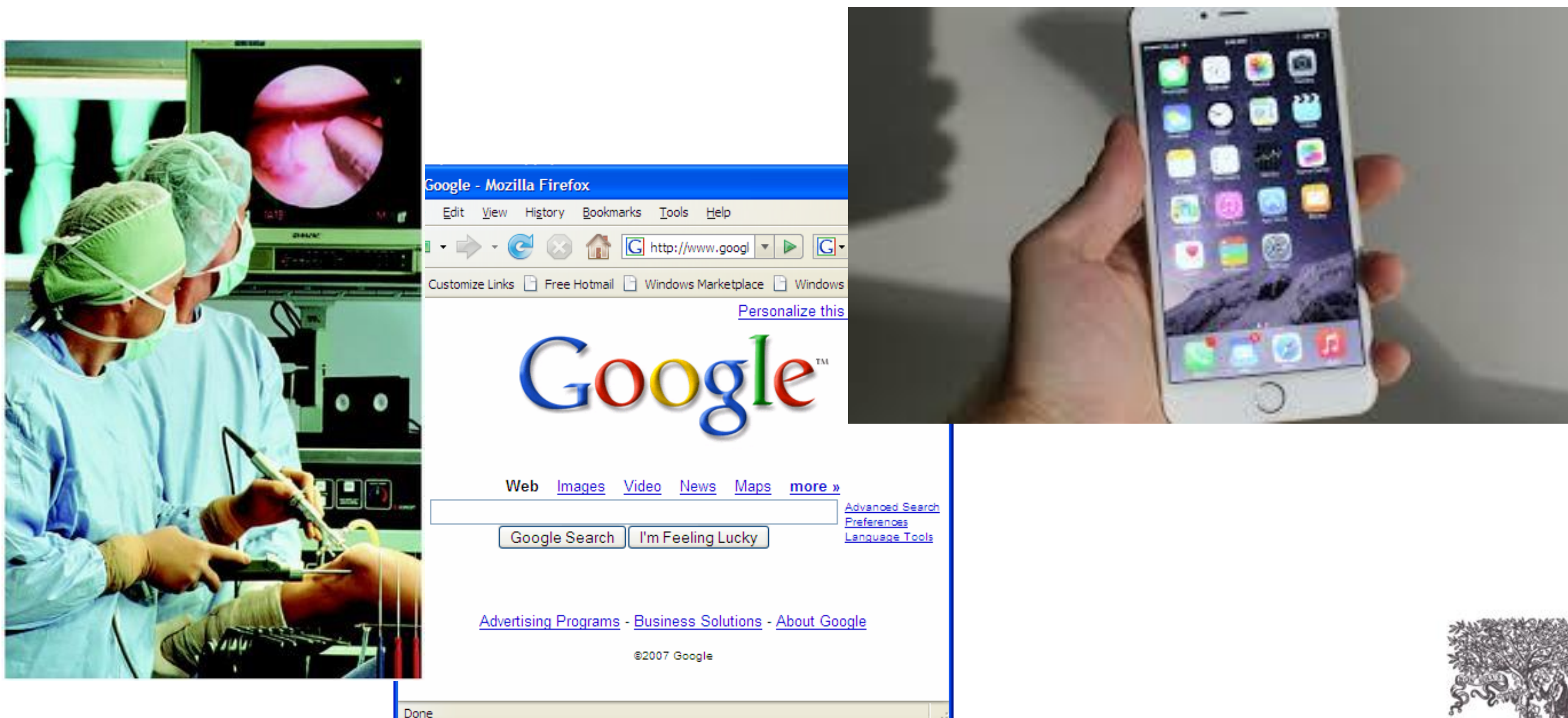
From Zero to One

Background: Digital Logic Design

- How have digital devices changed the world?
- How have digital devices changed *your* life?

Background

- Digital Devices have revolutionized our world
  - Internet, cell phones, rapid advances in medicine, etc.
- The semiconductor industry has grown from $21 billion in 1985 to over $300 billion in 2015



The Game Plan

- Purpose of course:
  - Learn the principles of digital design
  - Learn to systematically debug increasingly complex designs

Chapter 1: Topics

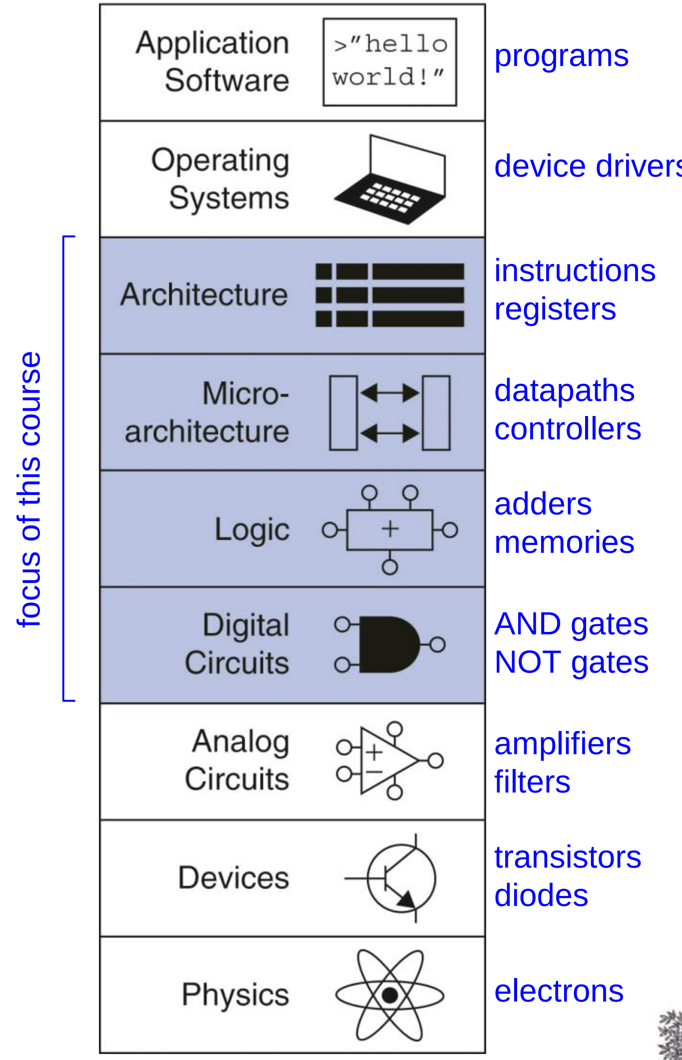
- The Art of Managing Complexity
- The Digital Abstraction
- Number Systems
- Addition
- Binary Codes
- Signed Numbers
- Logic Gates
- Logic Levels
- CMOS Transistors
- Power Consumption

The Art of Managing Complexity

- Abstraction
- Discipline
- The Three –y's
  - Hierarch**y**
  - Modularit**y**
  - Regularit**y**

Abstraction

- What is abstraction?
  - Hiding details when they are not important
- Electronic computer abstraction
  - Different levels with different building blocks



Discipline

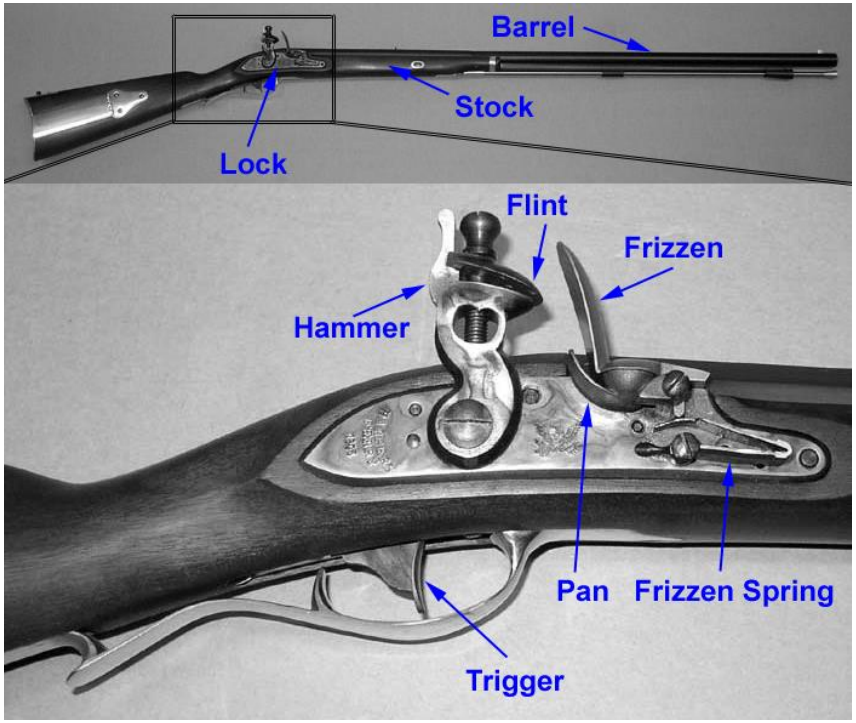
- Intentionally restrict design choices
- Example: Digital discipline
  - Discrete voltages (0 V, 5 V) instead of continuous (0V – 5V)
  - Simpler to design than analog circuits – can build more sophisticated systems
  - Digital systems replacing analog predecessors:
    - i.e., digital cameras, digital television, cell phones, CDs

The Three –y's

- Hierarchy
  - A system divided into modules and submodules
- Modularity
  - Having well-defined functions and interfaces
- Regularity
  - Encouraging uniformity, so modules can be easily reused

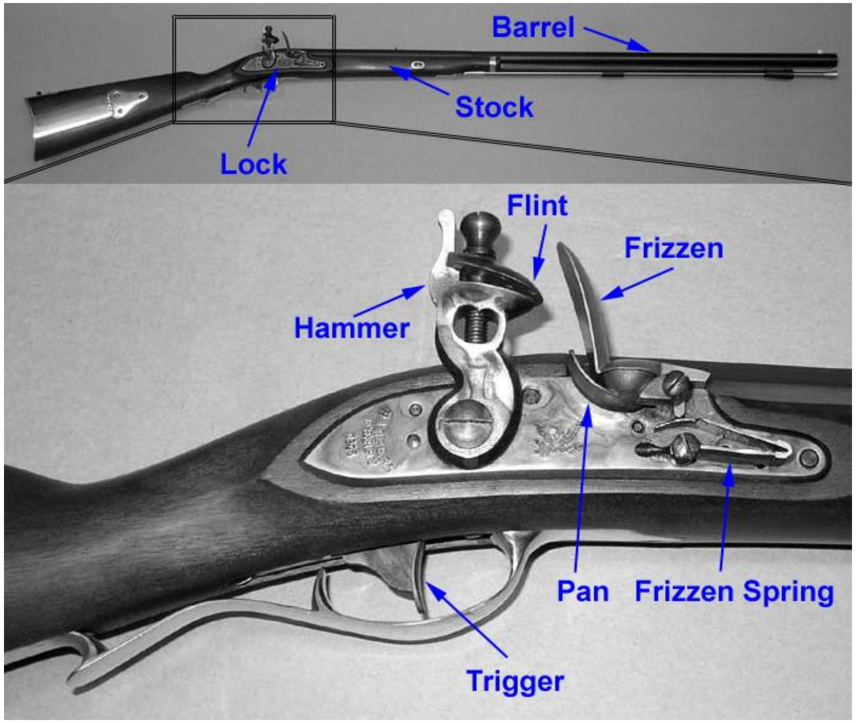
Example: Flintlock Rifle

- Hierarchy
  - Three main modules: Lock, stock, and barrel
  - Submodules of lock: Hammer, flint, frizzen, etc.



Example Flintlock Rifle

- Modularity
  - Function of stock: mount barrel and lock
  - Interface of stock: length and location of mounting pins
- Regularity
  - Interchangeable parts



The Art of Managing Complexity

- Abstraction
- Discipline
- The Three –y's
  - Hierarch**y**
  - Modularit**y**
  - Regularit**y**

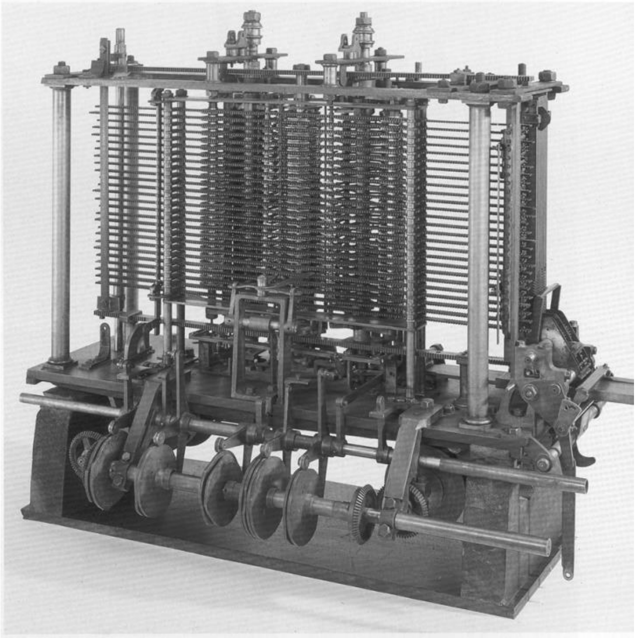
The Digital Abstraction

- Most physical variables are **continuous**
  - Voltage on a wire (1.33 V, 9 V, 12.2 V)
  - Frequency of an oscillation (60 Hz, 33.3 Hz, 44.1 kHz)
  - Position of mass (0.25 m, 3.2 m)
- Digital abstraction considers **discrete subset** of values
  - 0 V, 5 V
  - “0”, “1”

FROM ZERO TO ONE

The Analytical Engine

- Designed by Charles Babbage from 1834 – 1871
- Considered to be the first digital computer
- Built from mechanical gears, where each gear represented a discrete value (0-9)
- Babbage died before it was finished



Digital Discipline: Binary Values

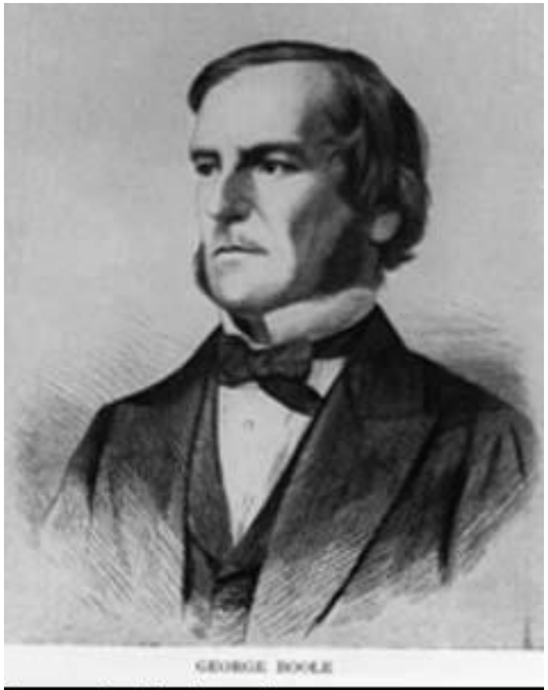
- Two discrete values
  - 1 and 0
    - 1 = TRUE = HIGH = ON
    - 0 = FALSE = LOW = OFF
- How to represent 1 and 0
  - Voltage levels, rotating gears, fluid levels, etc.
- Digital circuits use voltage levels to represent 1 and 0
  - Bit = binary digit
    - Represents the status of a digital signal (2 values)

Why Digital Systems?

- Easier to design
- Fast
- Can overcome noise
- Error detection/correction

George Boole, 1815-1864

- Born to working class parents
- Taught himself mathematics and joined the faculty of Queen's College in Ireland
- Wrote An Investigation of the Laws of Thought (1854)
- Introduced binary variables
- Introduced the three fundamental logic operations: AND, OR, and NOT



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Number Systems

- Decimal
  - Base 10
- Binary
  - Base 2
- Hexadecimal
  - Base 16

Decimal Numbers

- Base 10 (our everyday number system)

1000's Column
100's Column
10's Column
1's Column

$$5374_{10} = 5 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$$

Five Thousand | Three Hundred | Seven Tens | Four Ones

Base 10

Binary Numbers

- Base 2 (computer number system)

8's Column
4's Column
2's Column
1's Column

$$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

One Eight | One Four | Zero Two | One One

Base 2

Powers of Two

- $2^0 =$
- $2^1 =$
- $2^2 =$
- $2^3 =$
- $2^4 =$
- $2^5 =$
- $2^6 =$
- $2^7 =$
- $2^8 =$
- $2^9 =$
- $2^{10} =$
- $2^{11} =$
- $2^{12} =$
- $2^{13} =$
- $2^{14} =$
- $2^{15} =$

Powers of Two

- $2^0 = 1$
- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- $2^7 = 128$
- $2^8 = 256$
- $2^9 = 512$
- $2^{10} = 1024$
- $2^{11} = 2048$
- $2^{12} = 4096$
- $2^{13} = 8192$
- $2^{14} = 16384$
- $2^{15} = 32768$
- Handy to memorize up to 2^{10}

Bits, Bytes, Nibbles …

- Bits

10010110

most significant bit — least significant bit

- Bytes = 8 bits
- Nibble = 4 bits

byte

10010110

nibble

- Words = 32 bits
  - Hex digit to represent nibble

CEBF9AD7

most significant byte — least significant byte

Decimal to Binary Conversion

- Two Methods:
- Method 1: Find largest power of 2 that fits, subtract and repeat
- Method 2: Repeatedly divide by 2, remainder goes in next most significant bit

D2B: Method 1

- Find largest power of 2 that fits, subtract, repeat

53_{10}

D2B: Method 1

- Find largest power of 2 that fits, subtract, repeat

53_{10}	32×1
$53\text{-}32 = 21$	16×1
$21\text{-}16 = 5$	4×1
$5\text{-}4 = 1$	1×1

$$= 110101_2$$

D2B: Method 2

- Repeatedly divide by 2, remainder goes in next most significant bit

$$53_{10} =$$

D2B: Method 2

- Repeatedly divide by 2, remainder goes in next most significant bit

$53_{10} =$	$53/2 = 26$ R1	LSB
	$26/2 = 13$ R0	
	$13/2 = 6$ R1	
	$6/2 = 3$ R0	
	$3/2 = 1$ R1	
	$1/2 = 0$ R1	MSB

$$= 110101_2$$

Number Conversion

- Binary to decimal conversion
  - Convert 10011_2 to decimal

$$16 \times 1 + 8 \times 0 + 4 \times 0 + 2 \times 1 + 1 \times 1 = 19_{10}$$

- Decimal to binary conversion
  - Convert 47_{10} to binary

$$32 \times 1 + 16 \times 0 + 8 \times 1 + 4 \times 1 + 2 \times 1 + 1 \times 1 = 101111_2$$

D2B Example

- Convert 75_{10} to binary

D2B Example

- Convert 75_{10} to binary

$$75_{10} = 64 + 8 + 2 + 1 = 1001011_2$$

- Or

$75/2$	$= 37$	R1
$37/2$	$= 18$	R1
$18/2$	$= 9$	R0
$9/2$	$= 4$	R1
$4/2$	$= 2$	R0
$2/2$	$= 1$	R0
$1/2$	$= 0$	R1

Binary Values and Range

- N-digit decimal number
  - How many values?
  - Range?
- Example:
  3-digit decimal number
  - Possible values
  - Range

Binary Values and Range

- N-digit decimal number
  - How many values?
    - 10^N
  - Range?
    - $[0, 10^N - 1]$

- Example:
  3-digit decimal number
  - Possible values
    - $10^3 = 1000$
  - Range
    - $[0, 999]$

Binary Values and Range

- N-bit binary number
  - How many values?
  - Range?
- Example:
  3-bit binary number
  - Possible values
  - Range

Binary Values and Range

- N-bit binary number
  - How many values?
    - 2^N
  - Range?
    - $[0, 2^N - 1]$

- Example:
  3-bit binary number
  - Possible values
    - $2^3 = 8$
  - Range
    - $[0, 7] = [000_2, 111_2]$

Binary Values and Range

- N-digit decimal number
  - How many values?
    - 10^N
  - Range?
    - $[0, 10^N - 1]$
- Example:
  3-digit decimal number
  - Possible values
    - $10^3 = 1000$
  - Range
    - $[0, 999]$

- N-bit binary number
  - How many values?
    - 2^N
  - Range?
    - $[0, 2^N - 1]$
- Example:
  3-bit binary number
  - Possible values
    - $2^3 = 8$
  - Range
    - $[0, 7] = [000_2, 111_2]$

Hexadecimal Numbers

- Base 16 number system
- Shorthand for binary
  - Four binary digits (4-bit binary number) is a single hex digit

Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	
8	8	
9	9	
A	10	
B	11	
C	12	
D	13	
E	14	
F	15	

Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
  - Convert $4AF_{16}$ (also written 0x4AF) to binary

- Hexadecimal to decimal conversion:
  - Convert 0x4AF to decimal

Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
  - Convert $4AF_{16}$ (also written 0x4AF) to binary
    - 0x4AF $= 0100\ 1010\ 1111_2$

- Hexadecimal to decimal conversion:
  - Convert 0x4AF to decimal
    - $4 \times 16^2 + 10 \times 16^1 + 15 \times 16^0 = 1199_{10}$

Number Systems

- Popular
  - Decimal Base 10
  - Binary Base 2
  - Hexadecimal Base 16
- Others
  - Octal Base 8
  - Any other base

Octal Numbers

- Same as hex with one less binary digit

Octal Digit	Decimal Equivalent	Binary Equivalent
0	0	000
1	1	001
2	2	010
3	3	011
4	4	100
5	5	101
6	6	110
7	7	111

Number Systems

- In general, an N-digit number $\{a_{N-1}a_{N-2} \ldots a_1 a_0\}$ of base R in decimal equals
  - $a_{N-1}R^{N-1} + a_{N-2}R^{N-2} + \cdots + a_1R^1 + a_0R^0$
- Example: 4-digit $\{5173\}$ of base 8 (octal)

Number Systems

- In general, an N-digit number $\{a_{N-1}a_{N-2} \ldots a_1 a_0\}$ of base R in decimal equals
  - $a_{N-1}R^{N-1} + a_{N-2}R^{N-2} + \cdots + a_1 R^1 + a_0 R^0$
- Example: 4-digit $\{5173\}$ of base 8 (octal)
  - $5 \times 8^3 + 1 \times 8^2 + 7 \times 8^1 + 3 \times 8^0 = 2683_{10}$

FROM ZERO TO ONE

ELSEVIER

Decimal to Octal Conversion

- Remember two methods for D2B conversion
  - 1: remove largest multiple; 2: repeated divide
- Convert 29_{10} to octal

Decimal to Octal Conversion

- Remember two methods for D2B conversion
  - 1: remove largest multiple; 2: repeated divide
- Convert 29_{10} to octal
- Method 2

29/8	=3	R5	lsb
3/8	=0	R3	msb

$$29_{10} = 35_8$$

Decimal to Octal Conversion

- Remember two methods for D2B conversion
  - 1: remove largest multiple; 2: repeated divide
- Convert 29_{10} to octal
- Method 1

29   8×3=24

29-24=5

$$29_{10} = 24 + 5 = 3 \times 8^1 + 5 \times 8^0 = 35_8$$

  - Or (better scalability)

$$29_{10} = 16 + 8 + 4 + 1 = 11101_2 = 35_8$$

Octal to Decimal Conversion

- Convert 163_8 to decimal

ELSEVIER

Octal to Decimal Conversion

- Convert 163_8 to decimal

  - $163_8 = 1 \times 8^2 + 6 \times 8^1 + 3$
  - $163_8 = 64 + 48 + 3$
  - $163_8 = 115_{10}$

Recap: Binary and Hex Numbers

- Example 1: Convert 83_{10} to hex

- Example 2: Convert 01101011_2 to hex and decimal

- Example 3: Convert 0xCA3 to binary and decimal

Recap: Binary and Hex Numbers

- Example 1: Convert 83_{10} to hex
  - $83_{10} = 64 + 16 + 2 + 1 = 1010011_2$
  - $1010011_2 = 101\ 0011_2 = 53_{16}$

- Example 2: Convert 01101011_2 to hex and decimal
  - $01101011_2 = 0110\ 1011_2 = 6B_{16}$
  - 0x6B $= 6 \times 16^1 + 11 \times 16^0 = 96 + 11 = 107$

- Example 3: Convert 0xCA3 to binary and decimal
  - 0xCA3 $= 1100\ 1010\ 0011_2$
  - 0xCA3 $= 12 \times 16^2 + 10 \times 16^1 + 3 \times 16^0 = 3235_{10}$

Large Powers of Two

- 2^{10} = 1 kilo ≈ 1000 (1024)
- 2^{20} = 1 mega ≈ 1 million (1,048,576)
- 2^{30} = 1 giga ≈ 1 billion (1,073,741,824)
- 2^{40} = 1 tera ≈ 1 trillion (1,099,511,627,776)

FROM ZERO TO ONE

Large Powers of Two: Abbreviations

- 2^{10} = 1 kilo ≈ 1000 (1024)

  for example: 1 kB = 1024 Bytes

  1 kb = 1024 bits

- 2^{20} = 1 mega ≈ 1 million (1,048,576)

  for example: 1 MiB, 1 Mib (1 megabit)

- 2^{30} = 1 giga ≈ 1 billion (1,073,741,824)

  for example: 1 GiB, 1 Gib

Estimating Powers of Two

- What is the value of 2^{24}?

- How many values can a 32-bit variable represent?

Estimating Powers of Two

- What is the value of 2^{24}?
  - **$2^4 \times 2^{20} \approx$ 16 million**

- How many values can a 32-bit variable represent?
  - **$2^2 \times 2^{30} \approx$ 4 billion**

Binary Codes

Another way of representing decimal numbers in binary

Example binary codes:

- Weighted codes
  - Binary Coded Decimal (BCD) (8-4-2-1 code)
  - 6-3-1-1 code
  - 8-4-2-1 code (simple binary)
- Gray codes
- Excess-3 code
- 2-out-of-5 code

ASCII-Code

TABLE 1-3 ASCII Code

Character	ASCII Code							Character	ASCII Code							Character	ASCII Code						
	A_6	A_5	A_4	A_3	A_2	A_1	A_0		A_6	A_5	A_4	A_3	A_2	A_1	A_0		A_6	A_5	A_4	A_3	A_2	A_1	A_0
space	0	1	0	0	0	0	0	@	1	0	0	0	0	0	0	'	1	1	0	0	0	0	0
!	0	1	0	0	0	0	1	A	1	0	0	0	0	0	1	a	1	1	0	0	0	0	1
"	0	1	0	0	0	1	0	B	1	0	0	0	0	1	0	b	1	1	0	0	0	1	0
#	0	1	0	0	0	1	1	C	1	0	0	0	0	1	1	c	1	1	0	0	0	1	1
$	0	1	0	0	1	0	0	D	1	0	0	0	1	0	0	d	1	1	0	0	1	0	0
%	0	1	0	0	1	0	1	E	1	0	0	0	1	0	1	e	1	1	0	0	1	0	1
&	0	1	0	0	1	1	0	F	1	0	0	0	1	1	0	f	1	1	0	0	1	1	0
'	0	1	0	0	1	1	1	G	1	0	0	0	1	1	1	g	1	1	0	0	1	1	1
(	0	1	0	1	0	0	0	H	1	0	0	1	0	0	0	h	1	1	0	1	0	0	0
)	0	1	0	1	0	0	1	I	1	0	0	1	0	0	1	i	1	1	0	1	0	0	1
*	0	1	0	1	0	1	0	J	1	0	0	1	0	1	0	j	1	1	0	1	0	1	0
+	0	1	0	1	0	1	1	K	1	0	0	1	0	1	1	k	1	1	0	1	0	1	1
,	0	1	0	1	1	0	0	L	1	0	0	1	1	0	0	l	1	1	0	1	1	0	0
–	0	1	0	1	1	0	1	M	1	0	0	1	1	0	1	m	1	1	0	1	1	0	1
.	0	1	0	1	1	1	0	N	1	0	0	1	1	1	0	n	1	1	0	1	1	1	0
/	0	1	0	1	1	1	1	O	1	0	0	1	1	1	1	o	1	1	0	1	1	1	1
0	0	1	1	0	0	0	0	P	1	0	1	0	0	0	0	p	1	1	1	0	0	0	0
1	0	1	1	0	0	0	1	Q	1	0	1	0	0	0	1	q	1	1	1	0	0	0	1
2	0	1	1	0	0	1	0	R	1	0	1	0	0	1	0	r	1	1	1	0	0	1	0
3	0	1	1	0	0	1	1	S	1	0	1	0	0	1	1	s	1	1	1	0	0	1	1
4	0	1	1	0	1	0	0	T	1	0	1	0	1	0	0	t	1	1	1	0	1	0	0
5	0	1	1	0	1	0	1	U	1	0	1	0	1	0	1	u	1	1	1	0	1	0	1
6	0	1	1	0	1	1	0	V	1	0	1	0	1	1	0	v	1	1	1	0	1	1	0
7	0	1	1	0	1	1	1	W	1	0	1	0	1	1	1	w	1	1	1	0	1	1	1
8	0	1	1	1	0	0	0	X	1	0	1	1	0	0	0	x	1	1	1	1	0	0	0
9	0	1	1	1	0	0	1	Y	1	0	1	1	0	0	1	y	1	1	1	1	0	0	1
:	0	1	1	1	0	1	0	Z	1	0	1	1	0	1	0	z	1	1	1	1	0	1	0
;	0	1	1	1	0	1	1	[	1	0	1	1	0	1	1	{	1	1	1	1	0	1	1
<	0	1	1	1	1	0	0	\	1	0	1	1	1	0	0	|	1	1	1	1	1	0	0
=	0	1	1	1	1	0	1	]	1	0	1	1	1	0	1	}	1	1	1	1	1	0	1
>	0	1	1	1	1	1	0	^	1	0	1	1	1	1	0	~	1	1	1	1	1	1	0
?	0	1	1	1	1	1	1	—	1	0	1	1	1	1	1	delete	1	1	1	1	1	1	1

Binary Codes

Decimal #	8-4-2-1 (BCD)	6-3-1-1	Excess-3	2-out-of-5	Gray
0	0000	0000	0011	00011	0000
1	0001	0001	0100	00101	0001
2	0010	0011	0101	00110	0011
3	0011	0100	0110	01001	0010
4	0100	0101	0111	01010	0110
5	0101	0111	1000	01100	1110
6	0110	1000	1001	10001	1010
7	0111	1001	1010	10010	1011
8	1000	1011	1011	10100	1001
9	1001	1100	1100	11000	1000

Each code combination represents **a single decimal digit.**

Gray Codes

Decimal #	Gray
0	0000
1	0001
2	0011
3	0010
4	0110
5	1110
6	1010
7	1011
8	1001
9	1000

- Next number differs in only one bit position
  - **Example:** 000, 001, 011, 010, 110, 111, 101, 100
- **Example use:** Analog-to-Digital (A/D) converters. Changing 2 bits at a time (i.e., 011 →100) could cause large inaccuracies.
- Will use in K-maps

Addition

- Decimal

```
    3734
+   5168
--------
```

- Binary

```
    1011
+   0011
--------
```

Addition

- Decimal

```
     11  ← carries
   3734
+  5168
-------
   8902
```

- Binary

```
   1011
+  0011
-------
```

Addition

- Decimal

```
       11  ← carries
      3734
  +   5168
  ----------
      8902
```

- Binary

```
       11  ← carries
      1011
  +   0011
  ----------
      1110
```

Binary Addition Examples

- Add the following 4-bit binary numbers

$$\begin{array}{r} 1001 \\ +\ 0101 \\ \hline \end{array}$$

- Add the following 4-bit binary numbers

$$\begin{array}{r} 1011 \\ +\ 0110 \\ \hline \end{array}$$

Binary Addition Examples

- Add the following 4-bit binary numbers

```
      1
   1001
+  0101
-------
   1110
```

- Add the following 4-bit binary numbers

```
   1011
+  0110
-------
```

Binary Addition Examples

- Add the following 4-bit binary numbers

```
     1
  1001
+ 0101
------
  1110
```

- Add the following 4-bit binary numbers

```
  111
  1011
+ 0110
------
 10001
```

Overflow!

Overflow

- Digital systems operate on a **fixed number of bits**
- Overflow: when result is too big to fit in the available number of bits
- See previous example of 11 + 6

Signed Binary Numbers

- Sign/Magnitude Numbers
- Two's Complement Numbers

Sign/Magnitude

- 1 sign bit, *N*-1 magnitude bits
- Sign bit is the most significant (left-most) bit
  - **Positive number:** sign bit = 0
  - **Negative number:** sign bit = 1

$$A:\{a_{N-1}, a_{N-2}, \cdots a_2, a_1, a_0\}$$

$$A = (-1)^{a_{n-1}} \sum_{i=0}^{n-2} a_i 2^i$$

- Example, 4-bit sign/magnitude representations of ± 6:
  - +6 =
  - -6 =
- Range of an *N*-bit sign/magnitude number:
  -

Sign/Magnitude

- 1 sign bit, N-1 magnitude bits
- Sign bit is the most significant (left-most) bit
  - **Positive number:** sign bit = 0
  - **Negative number:** sign bit = 1

$$A:\{a_{N-1},a_{N-2},\cdots a_2,a_1,a_0\}$$

$$A=(-1)^{a_{n-1}}\sum_{i=0}^{n-2}a_i 2^i$$

- Example, 4-bit sign/magnitude representations of ± 6:
  - +6 = **0110**
  - -6 = **1110**
- Range of an N-bit sign/magnitude number:
  - **$[-(2^{N-1}-1), 2^{N-1}-1]$**

Sign/Magnitude Numbers

- Problems:
  - Addition doesn't work, for example -6 + 6:

$$\begin{array}{r} 1110 \\ +\ 0110 \\ \hline \end{array}$$

  - Two representations of 0 (± 0):
    - (+0) =
    - (−0) =

Sign/Magnitude Numbers

- Problems:
  - Addition doesn't work, for example -6 + 6:

$$\begin{array}{r} 1110 \\ +\ 0110 \\ \hline 10100 \end{array}$$ (wrong!)

  - Two representations of 0 (± 0):
    - $(+0) = 0000$
    - $(-0) = 1000$

Two's Complement Numbers

- Don't have same problems as sign/magnitude numbers:
  - Addition works
  - Single representation for 0

- Range of representable numbers not symmetric
  - One extra negative number

Two's Complement Numbers

- msb has value of -2^{N-1}

$$A = a_{n-1}\left(-2^{n-1}\right) + \sum_{i=0}^{n-2} a_i 2^i$$

- The most significant bit still indicates the sign (1 = negative, 0 = positive)
- Range of an *N*-bit two's comp number?

- Most positive 4-bit number?
- Most negative 4-bit number?

Two's Complement Numbers

- msb has value of -2^{N-1}

$$A = a_{n-1}\left(-2^{n-1}\right) + \sum_{i=0}^{n-2} a_i 2^i$$

- The most significant bit still indicates the sign (1 = negative, 0 = positive)
- Range of an *N*-bit two's comp number?
  - $[-(2^{N-1}), 2^{N-1} - 1]$
- Most positive 4-bit number? 0111
- Most negative 4-bit number? 1000

"Taking the Two's Complement"

- **Flips the sign** of a two's complement number
- Method:
  1. Invert the bits
  2. Add 1
- Example: Flip the sign of $3_{10} = 0011_2$

"Taking the Two's Complement"

- **Flips the sign** of a two's complement number
- Method:
  1. Invert the bits
  2. Add 1
- Example: Flip the sign of $3_{10} = 0011_2$
  1. **1100**
  2. **+ 1**

  $1101 = -3_{10}$

Two's Complement Examples

- Take the two's complement of $6_{10} = 0110_2$

- What is the decimal value of the two's complement number 1001_2?

Two's Complement Examples

- Take the two's complement of $6_{10} = 0110_2$
  1. 1001
  2. + 1

  $1010_2 = -6_{10}$

- What is the decimal value of the two's complement number 1001_2?
  1. 0110
  2. + 1

  $0111_2 = 7_{10}$, so $1001_2 = -7_{10}$

FROM ZERO TO ONE

ELSEVIER

Two's Complement Addition

- Add 6 + (-6) using two's complement numbers

```
     0110
+    1010
---------
```

- Add -2 + 3 using two's complement numbers

```
     1110
+    0011
---------
```

Two's Complement Addition

- Add 6 + (-6) using two's complement numbers

```
 111
   0110
+  1010
-------
  10000
```

- Add -2 + 3 using two's complement numbers

```
   1110
+  0011
-------
```

Two’s Complement Addition

- Add 6 + (-6) using two’s complement numbers

```
   111
    0110
+   1010
--------
   10000
```

- Add -2 + 3 using two’s complement numbers

```
   111
    1110
+   0011
--------
   10001
```

Increasing Bit Width

- Extend number from N to M bits (M > N) :
  - Sign-extension
  - Zero-extension

Sign-Extension

- Sign bit copied to msb's
- Number value is same

- Example 1
  - 4-bit representation of 3 = 0011
  - 8-bit sign-extended value:
- Example 2
  - 4-bit representation of -7 = 1001
  - 8-bit sign-extended value:

Sign-Extension

- Sign bit copied to msb's
- Number value is same

- Example 1
  - 4-bit representation of 3 = 0011
  - 8-bit sign-extended value: 00000011
- Example 2
  - 4-bit representation of -7 = 1001
  - 8-bit sign-extended value: 11111001

Zero-Extension

- Zeros copied to msb's
- Value changes for negative numbers

- Example 1
  - 4-bit value = 0011_2
  - 8-bit zero-extended value:
- Example 2
  - 4-bit value = 1001
  - 8-bit zero-extended value:

Zero-Extension

- Zeros copied to msb's
- Value changes for negative numbers

- Example 1
  - 4-bit value = 0011_2
  - 8-bit zero-extended value: 00000011
- Example 2
  - 4-bit value = 1001
  - 8-bit zero-extended value: 00001001

Zero-Extension

- Zeros copied to msb's
- Value changes for negative numbers

- Example 1
  - 4-bit value = $0011_2 = 3_{10}$
  - 8-bit zero-extended value: $00000011 = 3_{10}$
- Example 2
  - 4-bit value = $1001 = -7_{10}$
  - 8-bit zero-extended value: $00001001 = 9_{10}$

Number System Comparison

Number System	Range
Unsigned	$[0, 2^{N}-1]$
Sign/Magnitude	$[-(2^{N-1}-1), 2^{N-1}-1]$
Two's Complement	$[-2^{N-1}, 2^{N-1}-1]$

For example, 4-bit representation:

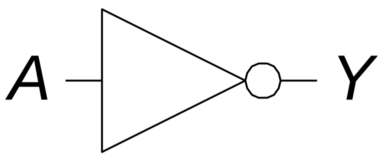
Value	Unsigned	Two's Complement	Sign/Magnitude
-8		1000	
-7		1001	1111
-6		1010	1110
-5		1011	1101
-4		1100	1100
-3		1101	1011
-2		1110	1010
-1		1111	1001
0	0000	0000	0000 1000
1	0001	0001	0001
2	0010	0010	0010
3	0011	0011	0011
4	0100	0100	0100
5	0101	0101	0101
6	0110	0110	0110
7	0111	0111	0111
8	1000		
9	1001		
10	1010		
11	1011		
12	1100		
13	1101		
14	1110		
15	1111		

Logic Gates

- **Perform logic functions:**
  - inversion (NOT), AND, OR, NAND, NOR, etc.
- **Single-input:**
  - NOT gate, buffer
- **Two-input:**
  - AND, OR, XOR, NAND, NOR, XNOR
- **Multiple-input**

Single-Input Logic Gates

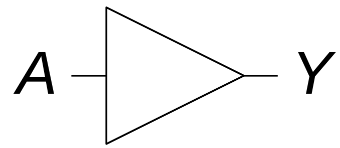
NOT



$Y = \overline{A}$

A	Y
0	
1	

BUF



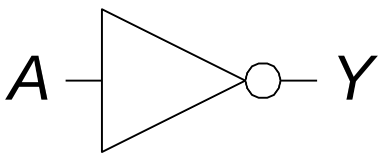
$Y = A$

A	Y
0	
1	

Single-Input Logic Gates

- Bubble on wire indicates inversion

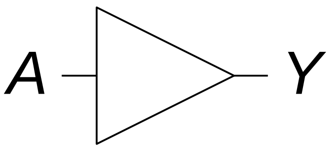
NOT



$Y = \overline{A}$

A	Y
0	1
1	0

BUF



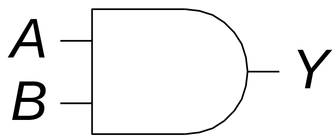
$Y = A$

A	Y
0	0
1	1

- Note: bar over variable indicates complement (invert value)

Two-Input Logic Gates

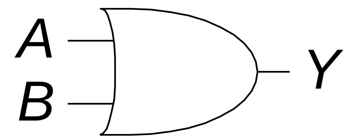
AND



$Y = AB$

A	B	Y
0	0	
0	1	
1	0	
1	1	

OR

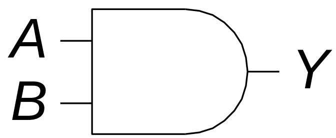


$Y = A + B$

A	B	Y
0	0	
0	1	
1	0	
1	1	

Two-Input Logic Gates

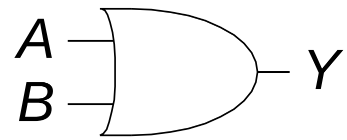
AND



$$Y = AB$$

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR

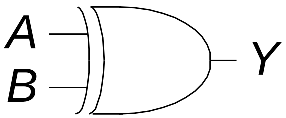


$$Y = A + B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

More Two-Input Logic Gates

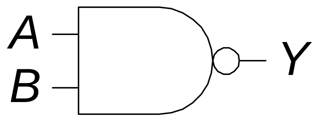
XOR



$$Y = A \oplus B$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

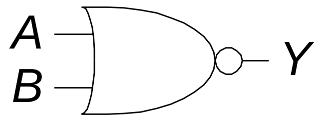
NAND



$$Y = \overline{AB}$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

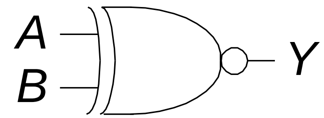
NOR



$$Y = \overline{A + B}$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

XNOR

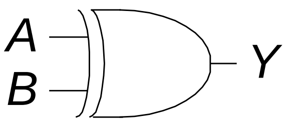


$$Y = \overline{A \oplus B}$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

More Two-Input Logic Gates

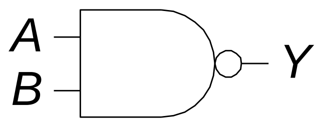
XOR



$$Y = A \oplus B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

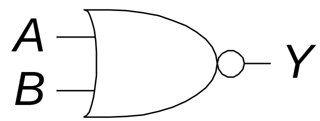
NAND



$$Y = \overline{AB}$$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

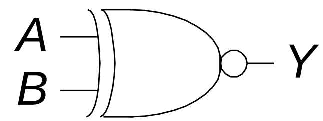
NOR



$$Y = \overline{A + B}$$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

XNOR

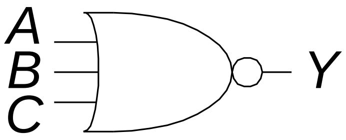


$$Y = \overline{A \oplus B}$$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Multiple-Input Logic Gates

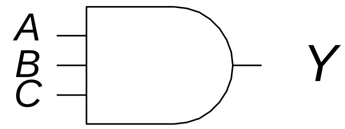
NOR3



$$Y = \overline{A+B+C}$$

A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

AND3

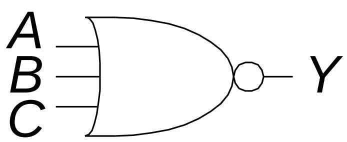


$$Y = ABC$$

A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Multiple-Input Logic Gates

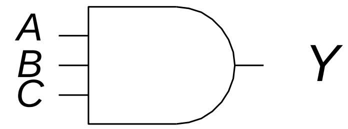
NOR3



$Y = \overline{A+B+C}$

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

AND3



$Y = ABC$

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

- Multi-input XOR = Odd parity (#on inputs odd→1)

Logic Levels

- Discrete voltages represent 1 and 0
- For example:
  - 0 = *ground* (GND) or 0 volts
  - 1 = V_{DD} or 5 volts
- What about 4.99 volts? Is that a 0 or a 1?
- What about 3.2 volts?

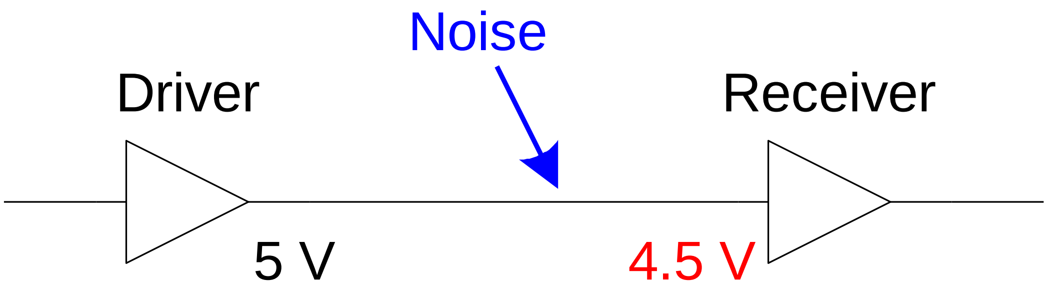
Logic Levels

- Must have *range* of voltages for 1 and 0
- Different ranges for inputs and outputs to allow for *noise*

What is Noise?

- Anything that degrades the signal
  - E.g., resistance, power supply noise, coupling to neighboring wires, etc.

- Example: a gate (driver) outputs 5 V but, because of resistance in a long wire, receiver gets 4.5 V



The Static Discipline

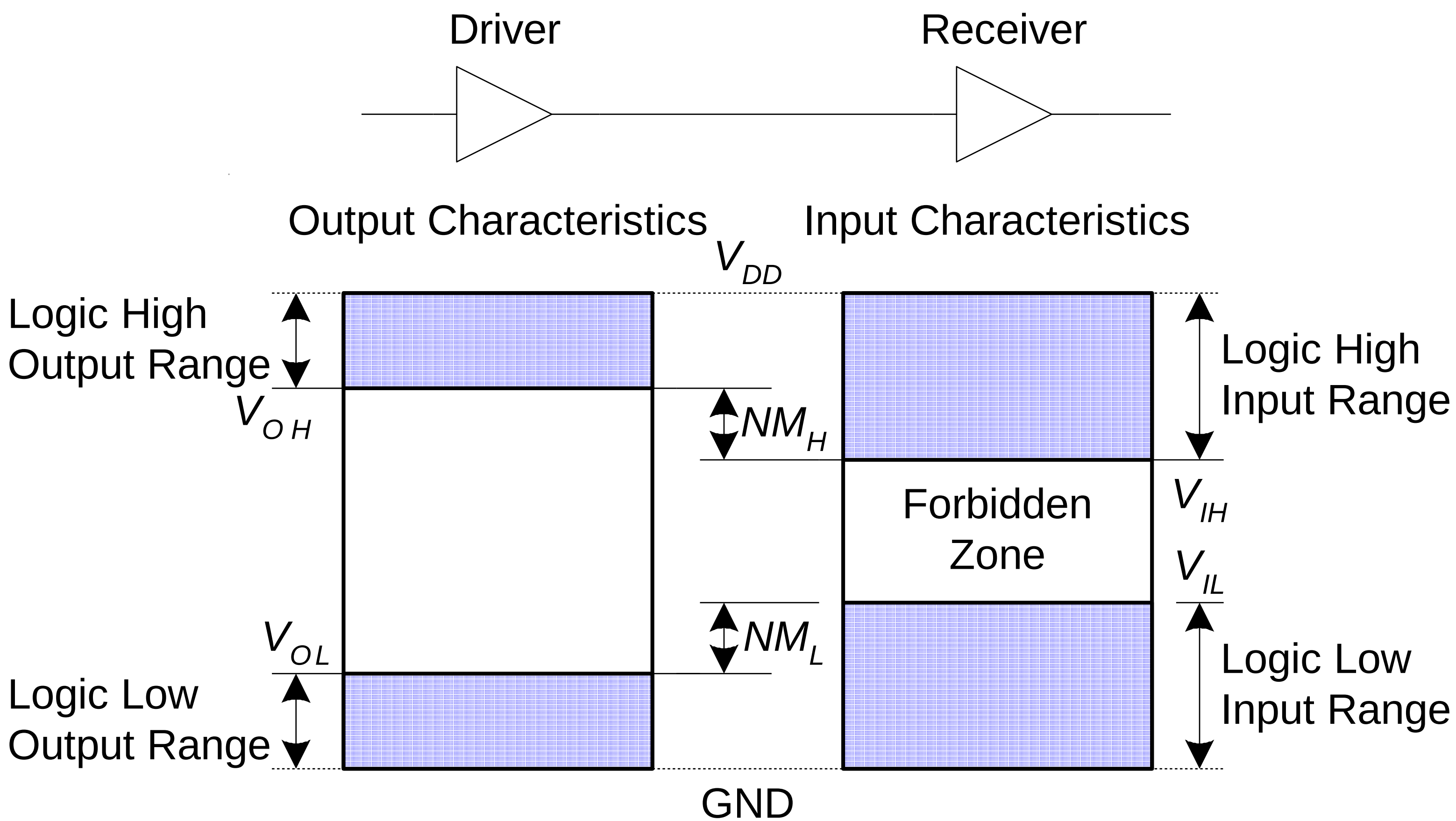
- With logically valid inputs, every circuit element must produce logically valid outputs

- Use limited ranges of voltages to represent discrete values

Real Logic Levels

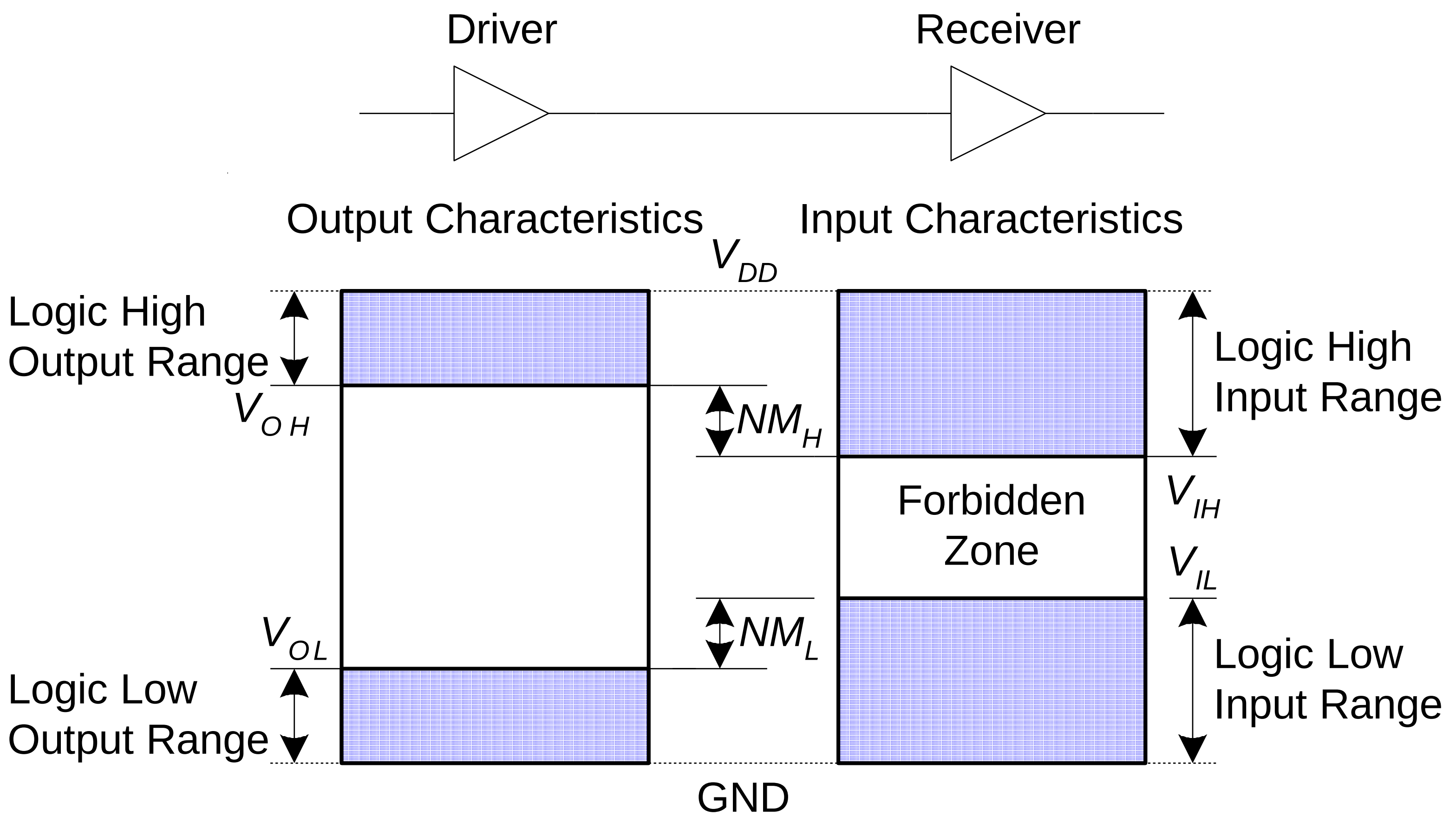
- Want driver to output "clean" high/low and receiver to handle noisy high/low

Real Logic Levels



- Want driver to output "clean" high/low and receiver to handle noisy high/low

Real Logic Levels



$$NM_H = V_{OH} - V_{IH}$$

$$NM_L = V_{IL} - V_{OL}$$

V_{DD} Scaling

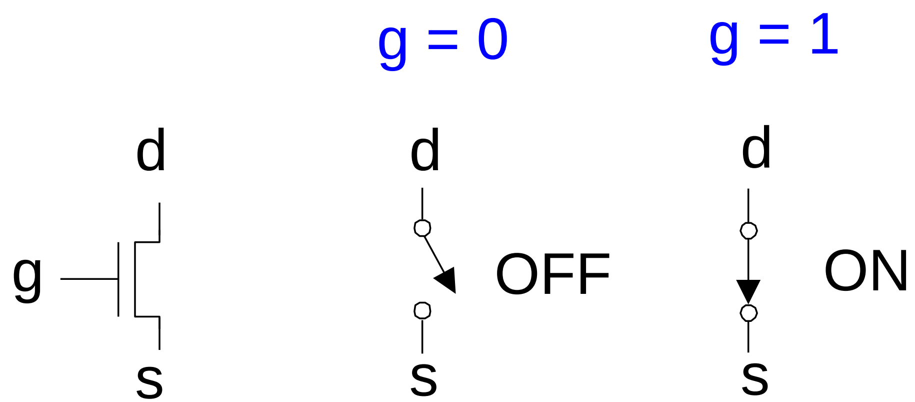
- In 1970's and 1980's, $V_{DD} = 5$ V
- V_{DD} has dropped
  - 3.3 V, 2.5 V, 1.8 V, 1.5 V, 1.2 V, 1.0 V, …
  - Avoid frying tiny transistors
  - Save power
- Be careful connecting chips with different supply voltages
  - Easy to fry if not careful

Logic Family Examples

Logic Family	V_{DD}	V_{IL}	V_{IH}	V_{OL}	V_{OH}
TTL	5 (4.75 - 5.25)	0.8	2.0	0.4	2.4
CMOS	5 (4.5 - 6)	1.35	3.15	0.33	3.84
LVTTL	3.3 (3 - 3.6)	0.8	2.0	0.4	2.4
LVCMOS	3.3 (3 - 3.6)	0.9	1.8	0.36	2.7

Transistors

- Logic gates built from transistors
- Simple model: 3-ported voltage-controlled switch
  - 2 ports connected depending on voltage of 3rd
  - d and s are connected (ON) when g is 1

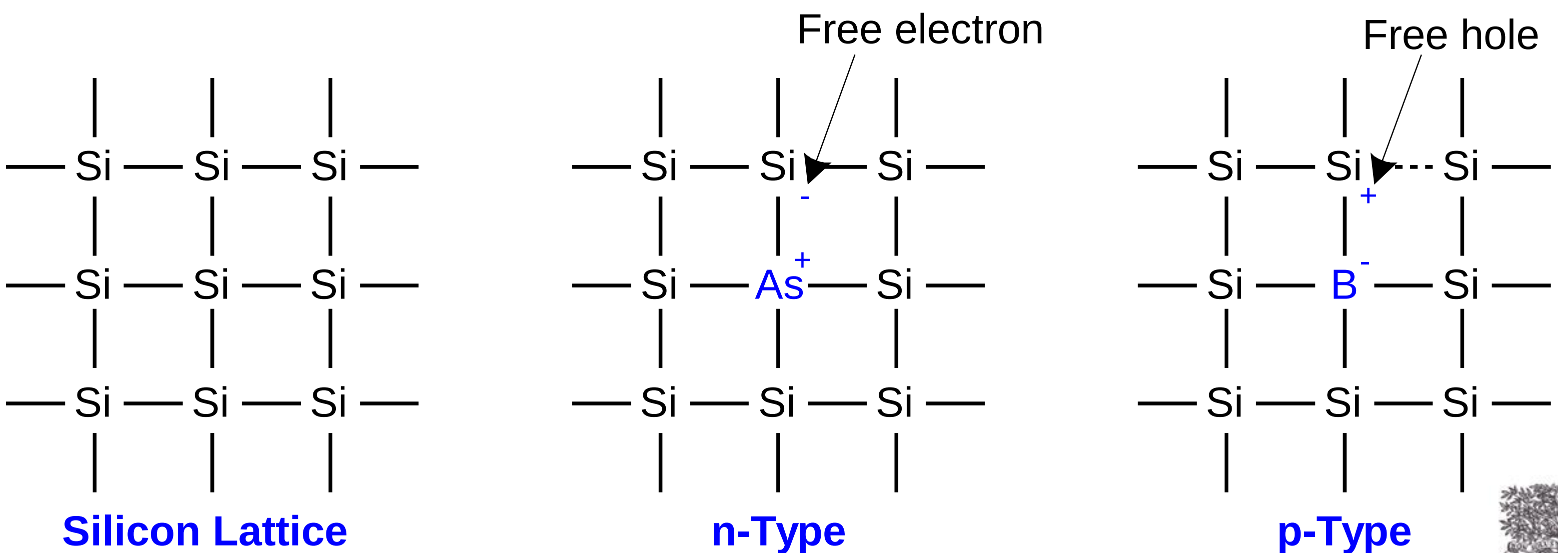


Robert Noyce, 1927-1990

- Nicknamed “Mayor of Silicon Valley”
- Cofounded Fairchild Semiconductor in 1957
- Cofounded Intel in 1968
- Co-invented the integrated circuit

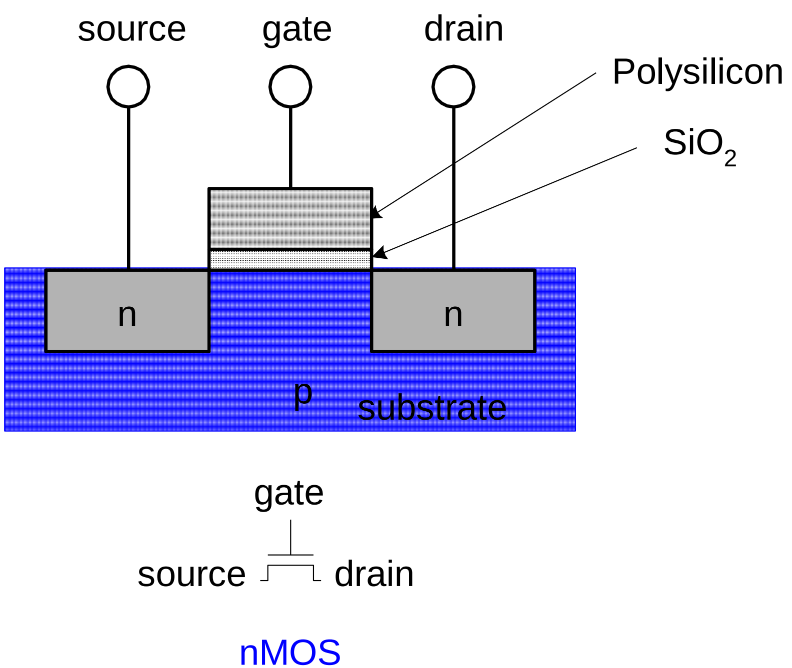
Silicon

- Transistors built from silicon, a semiconductor
- Pure silicon is a poor conductor (no free charges)
- Doped silicon is a good conductor (free charges)
  - n-type (free negative charges, electrons)
  - p-type (free positive charges, holes)



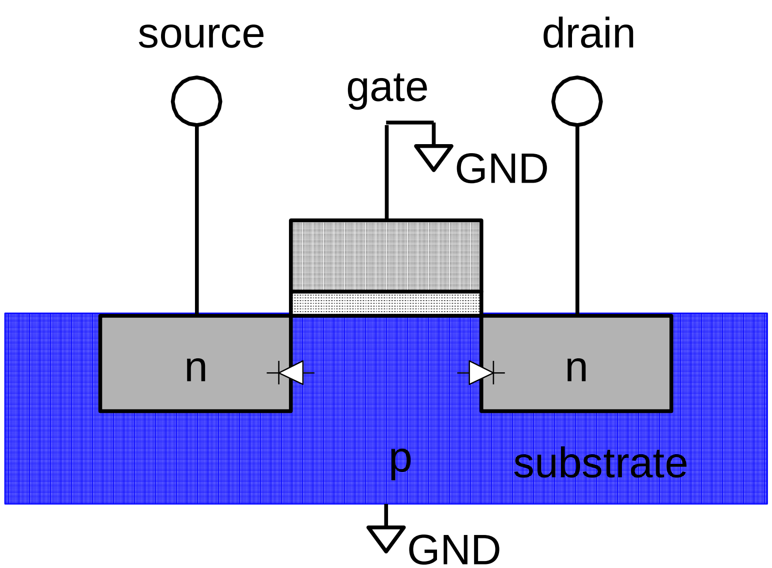
MOS Transistors

- Metal oxide silicon (MOS) transistors:
  - Polysilicon (used to be metal) gate
  - Oxide (silicon dioxide) insulator
  - Doped silicon

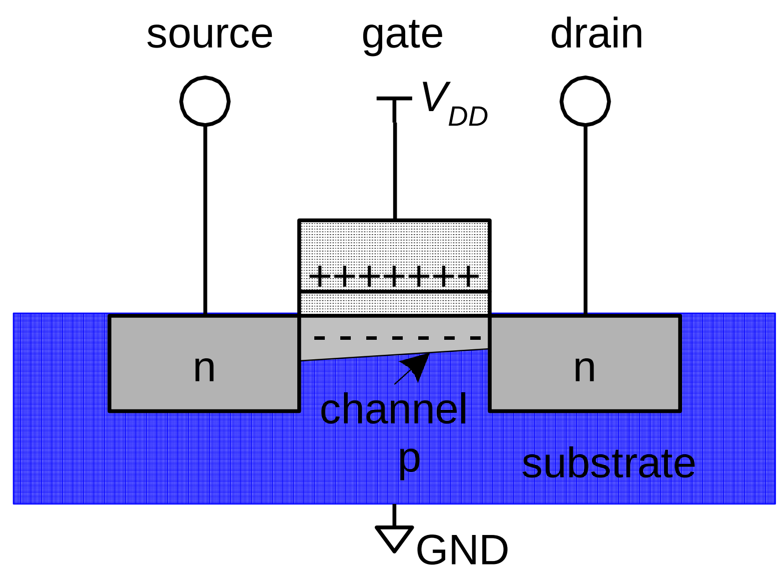


nMOS Transistors

- Gate = 0
- OFF (no connection between source and drain)



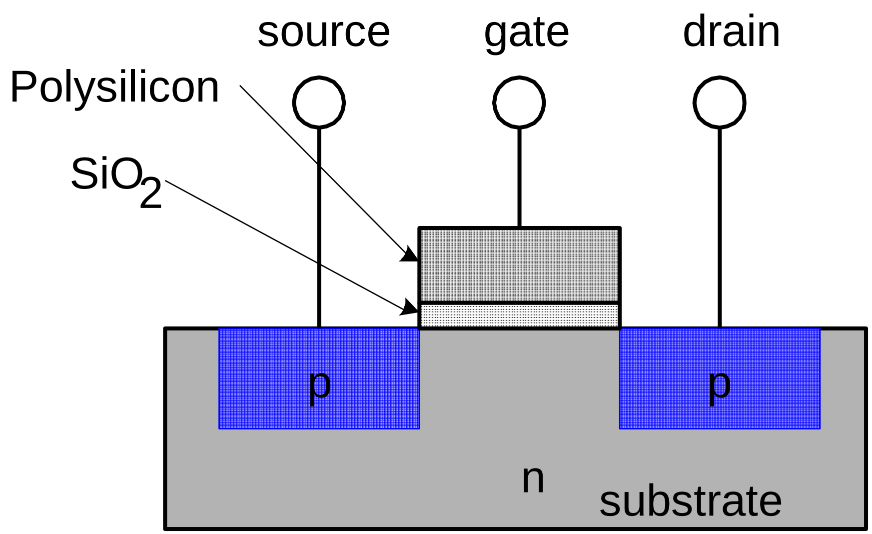
- Gate = 1
- ON (channel between source and drain)



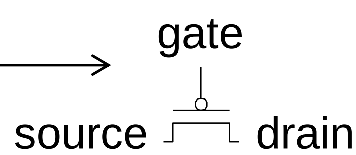
Diode connection from p to n doped area
→ current cannot travel from n→p

pMOS Transistors

- pMOS transistor is opposite of nMOS
  - ON when Gate = 0
  - OFF when Gate = 1

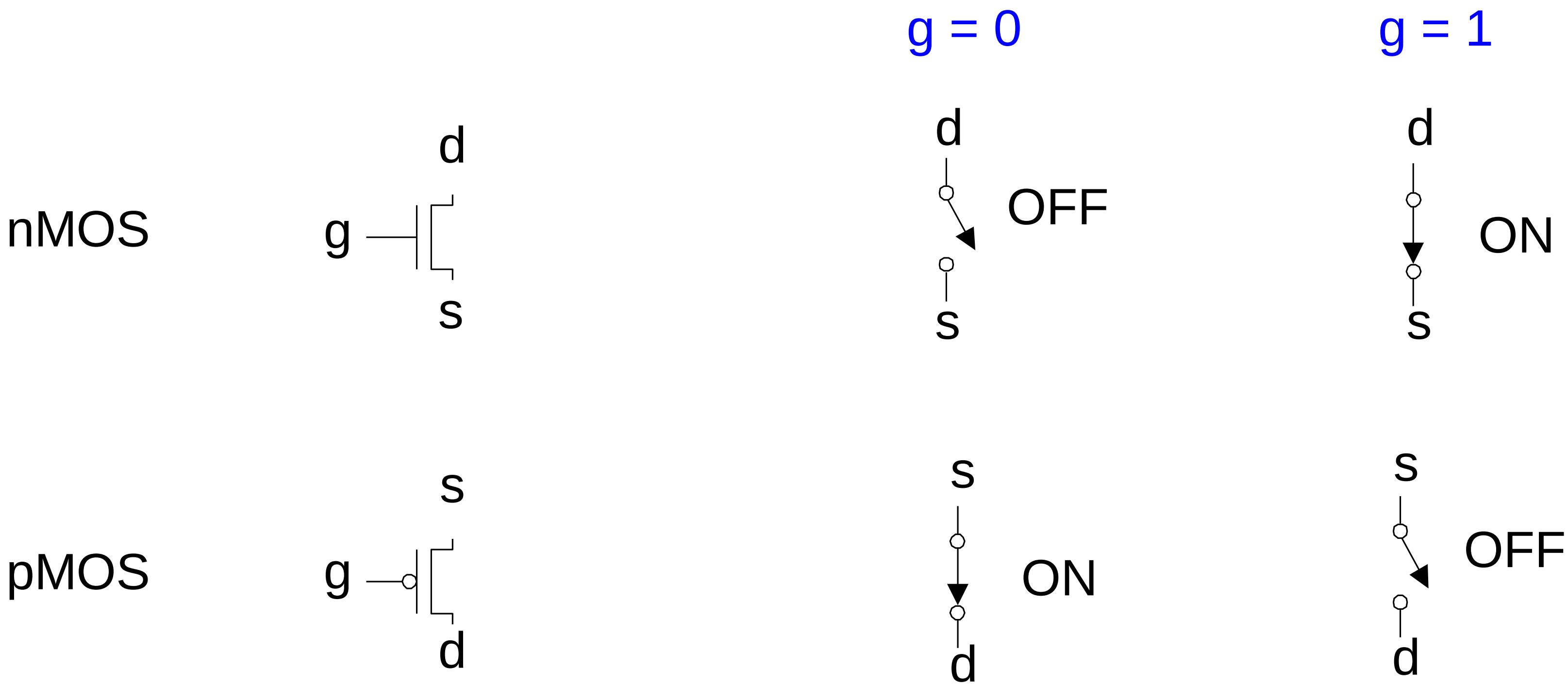


Note bubble on gate
to indicate on when low



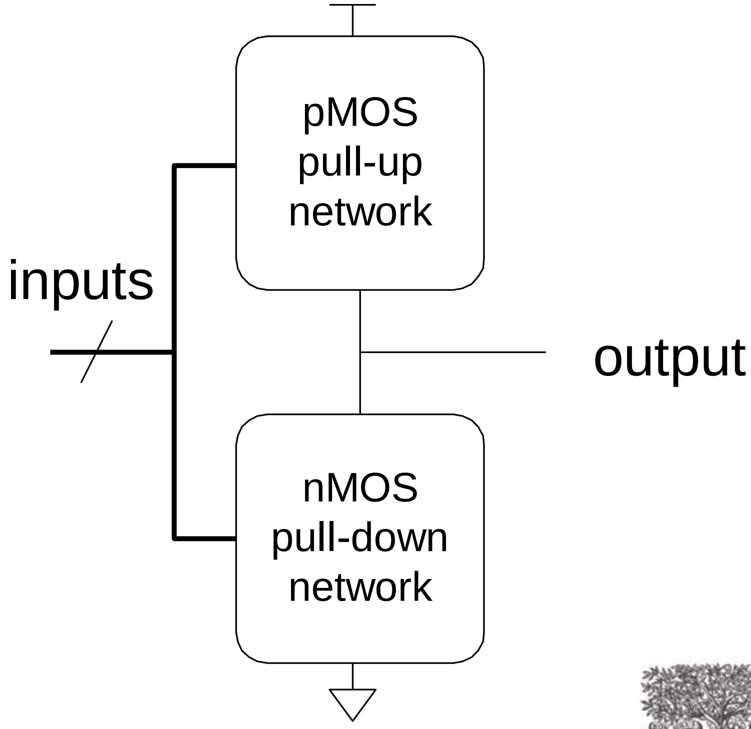
Transistor Function

- Voltage controlled switch



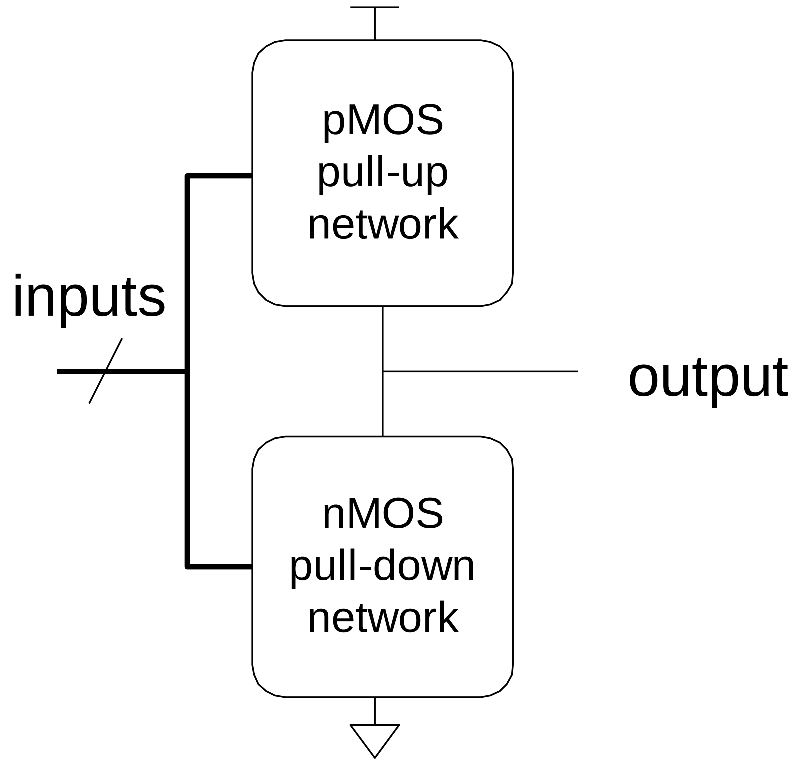
Transistor Composition

- nMOS: pass good 0's
  - Connect source to GND
  - "Pull down" transistor
- pMOS: pass good 1's
  - Connect source to VDD
  - "Pull up" transistor
- Build logic gates from composition
  - CMOS = complementary MOS



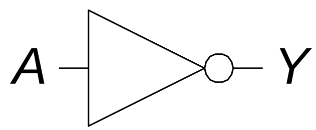
CMOS Gate Structure

- Pull-up pMOS network connects to V_{DD}
- Pull-down nMOS network connects to GND
- Use series and parallel connections to implement gate logic



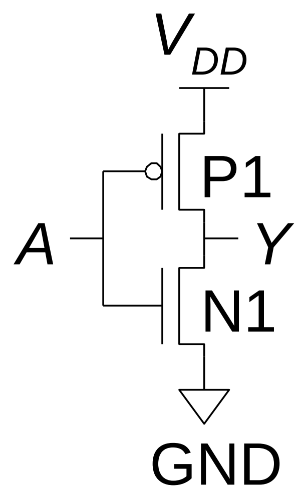
CMOS Gates: NOT Gate

NOT



$Y = \overline{A}$

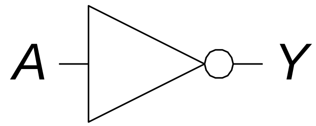
A	Y
0	1
1	0



A	P1	N1	*Y*
0			
1			

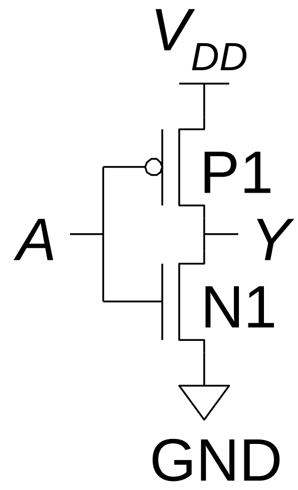
CMOS Gates: NOT Gate

NOT



$Y = \overline{A}$

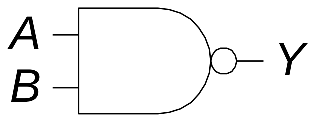
A	Y
0	1
1	0



A	P1	N1	Y
0	ON	OFF	1
1	OFF	ON	0

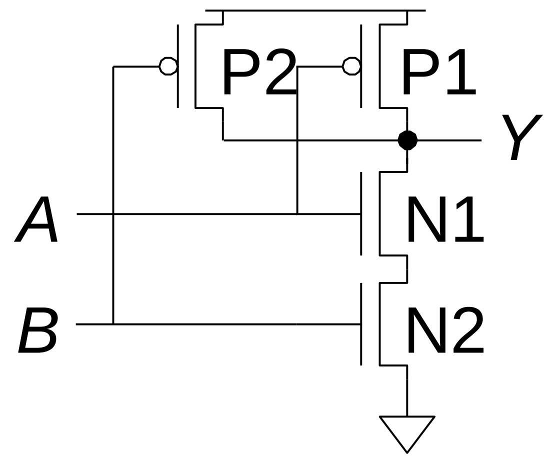
CMOS Gates: NAND Gate

NAND



$Y = \overline{AB}$

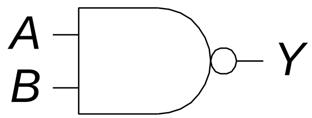
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



A	B	P1	P2	N1	N2	Y
0	0					
0	1					
1	0					
1	1					

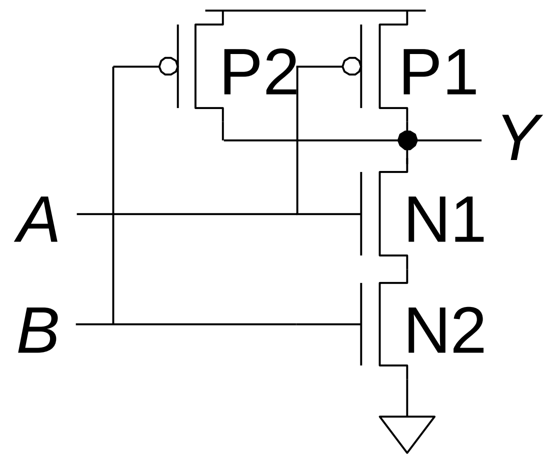
CMOS Gates: NAND Gate

NAND



$Y = \overline{AB}$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



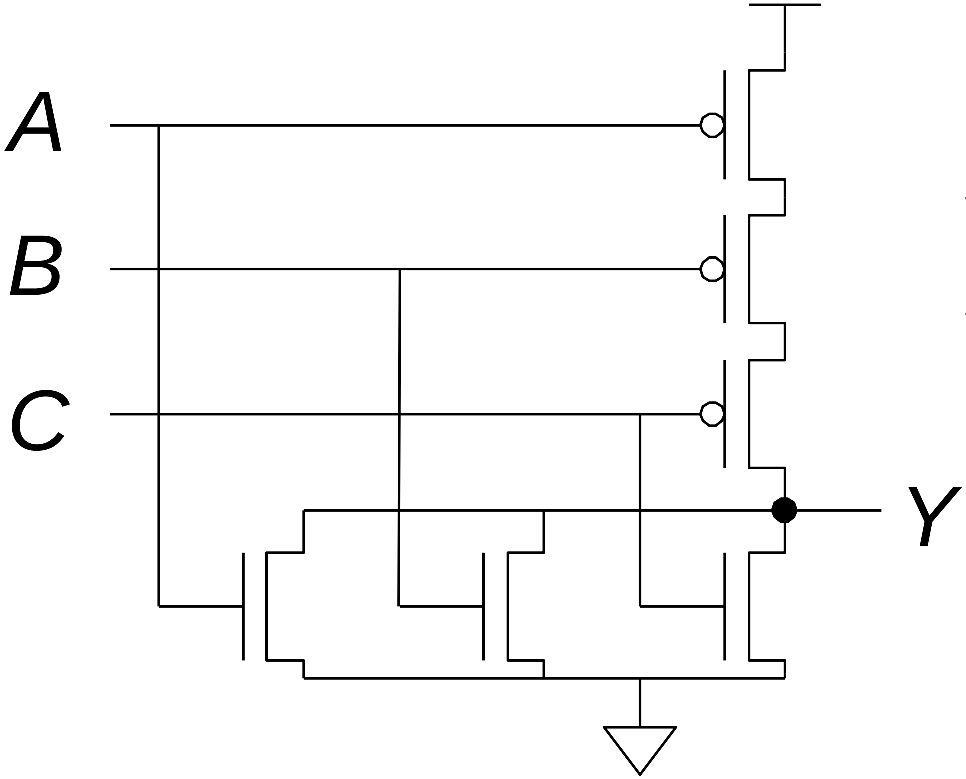
A	*B*	P1	P2	N1	N2	*Y*
0	0	ON	ON	OFF	OFF	1
0	1	ON	OFF	OFF	ON	1
1	0	OFF	ON	ON	OFF	1
1	1	OFF	OFF	ON	ON	0

CMOS Gates: NOR Gate

- How can you build three input (A, B, C)NOR gate?

CMOS Gates: NOR Gate

- How can you build three input (A, B, C)NOR gate?



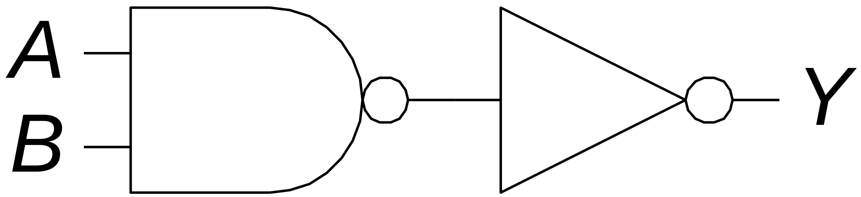
Only high output when all three pMOS in series are "on" and create a path from output to V_{DD}

CMOS Gates: AND Gate

- How can you build 2 input AND gate?

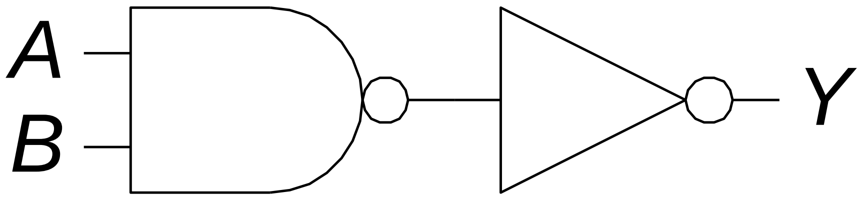
CMOS Gates: AND Gate

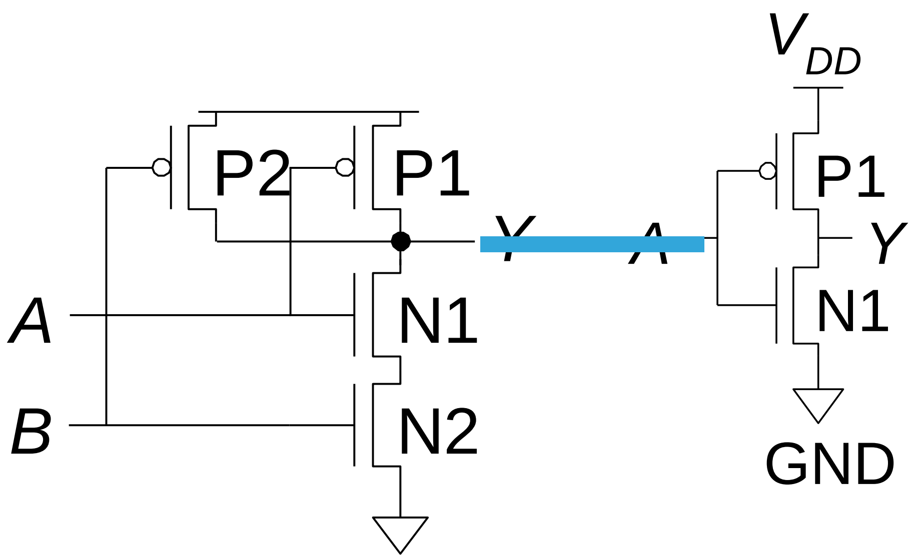
- How can you build 2 input AND gate?



CMOS Gates: AND Gate

- How can you build 2 input AND gate?

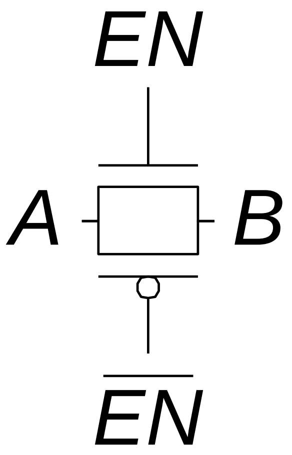




Note: AND requires 2 more gates than NAND. Inverted logic is more efficient implementation.

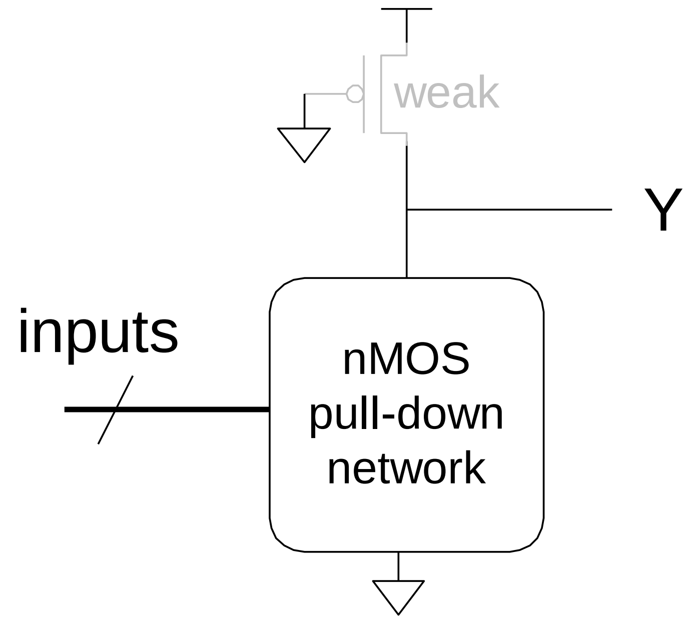
Transmission Gates

- nMOS pass 1's poorly, pMOS pass 0's poorly
- Transmission gate is for passing signal
  - Pass both 0 and 1 well
- When EN = 1, the switch is ON:
  - $\overline{EN} = 0$ and A is connected to B
- When EN = 0, the switch is OFF:
  - A is not connected to B



Psuedo-nMOS

- Replace pull-up network with weak pMOS transistor that is always on
  - pMOS gate tied to ground
- pMOS transistor: pulls output HIGH only when nMOS network not pulling it LOW

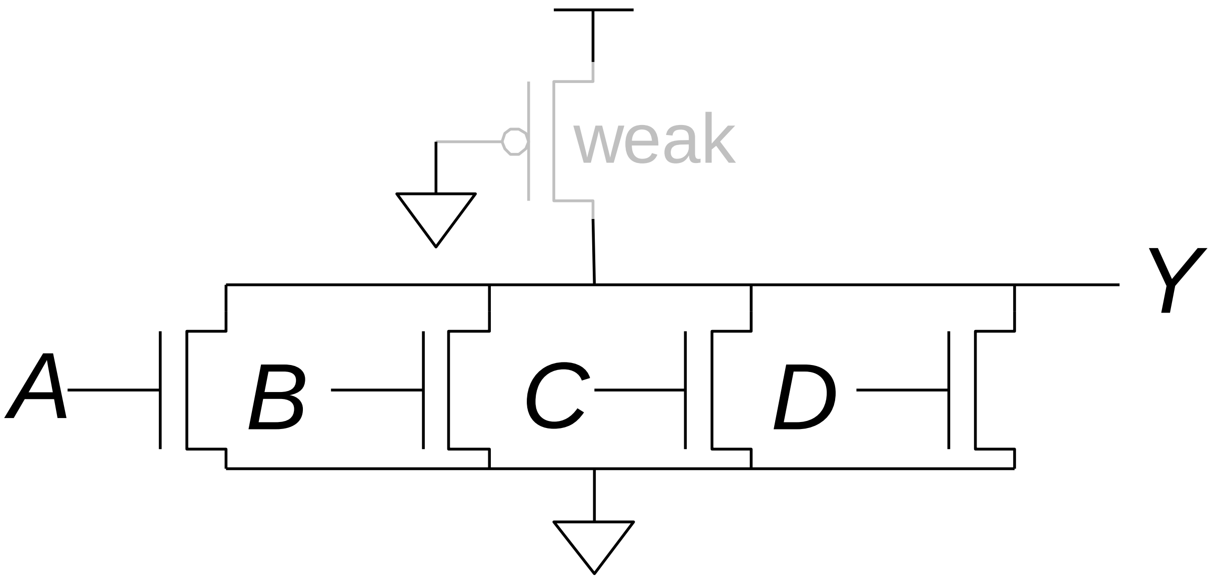


Psuedo-nMOS Example: NOR4

- How many transistors needed?

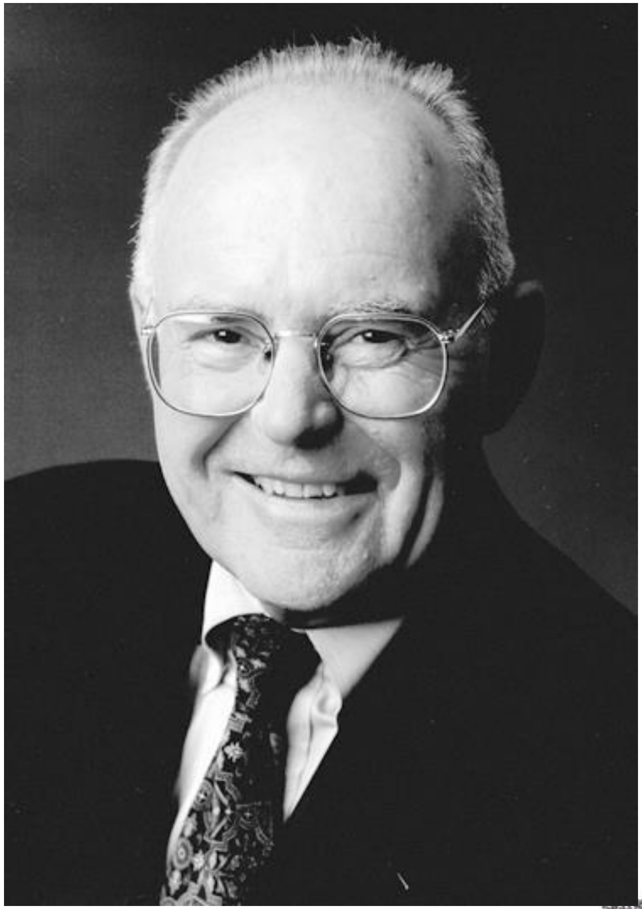
Psuedo-nMOS Example: NOR4

- How many transistors needed?
  - Only 5 since a single pMOS is used



Gordon Moore, 1929-

- Cofounded Intel in 1968 with Robert Noyce.
- Moore's Law: number of transistors on a computer chip doubles every year (observed in 1965)
  - Since 1975, transistor counts have doubled every two years.

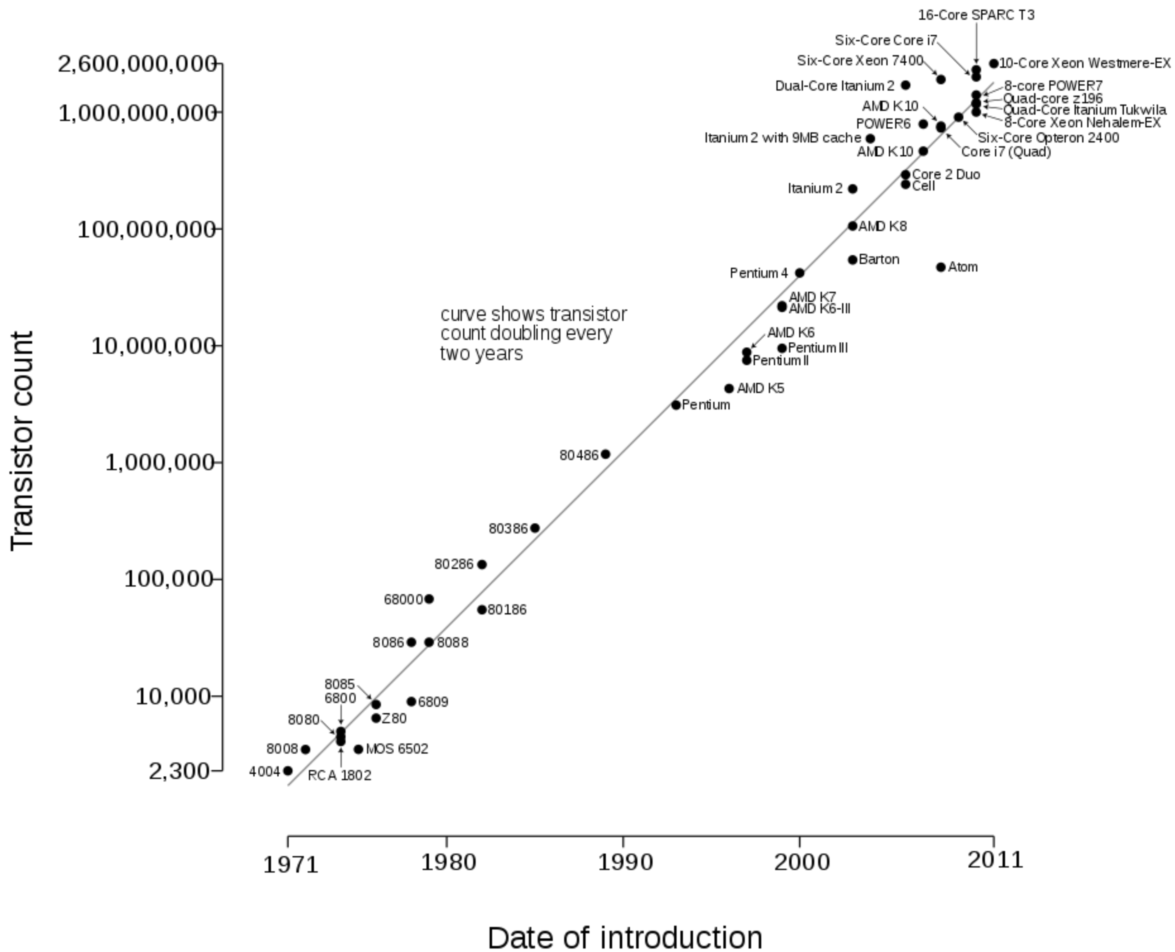


FROM ZERO TO ONE

Moore’s Law

- Transistor count doubles every 2 years

Microprocessor Transistor Counts 1971-2011 & Moore's Law



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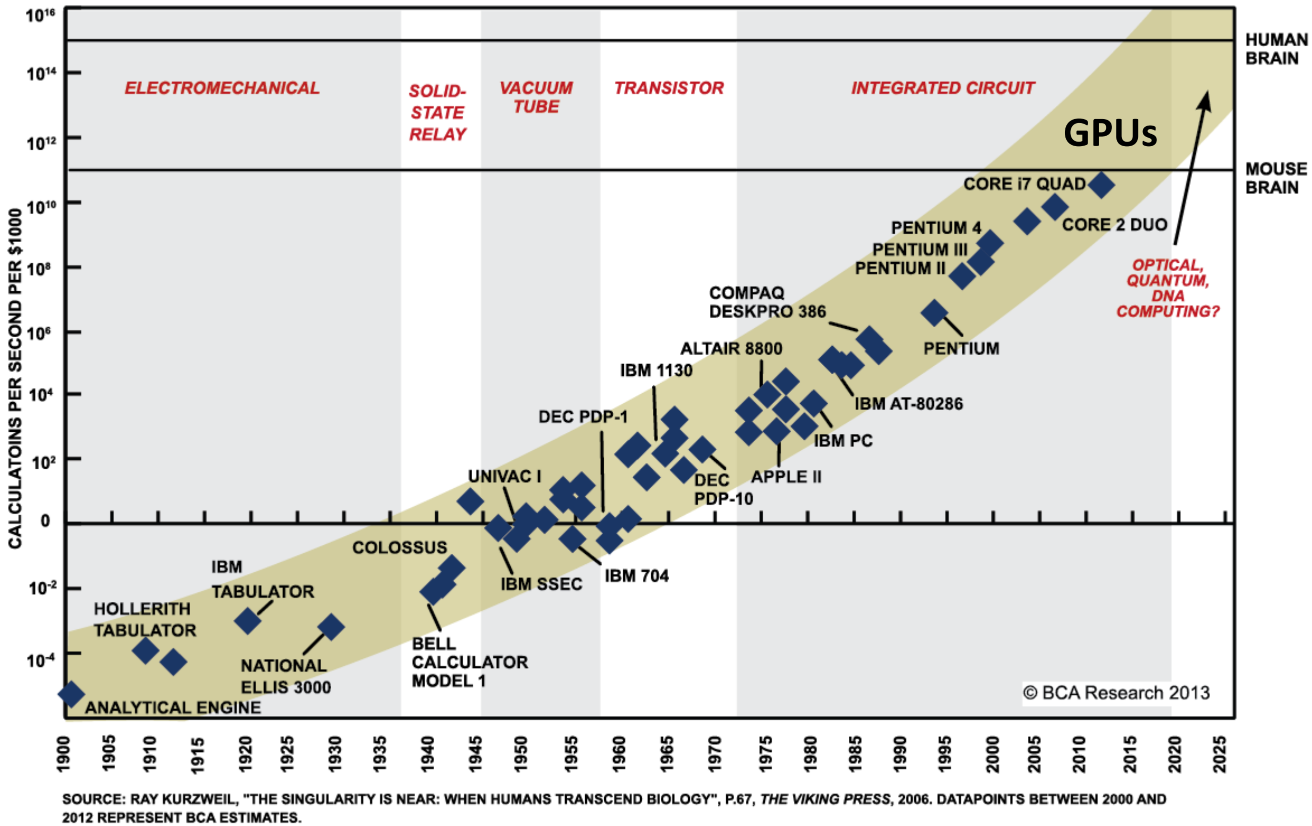
Moore's Law Trends



- *"If the automobile had followed the same development cycle as the computer, a Rolls-Royce would today cost $100, get one million miles to the gallon, and explode once a year . . ."*

  – Robert Cringley

Power Consumption

- Power = Energy consumed per unit time

- Two types of power
  - Dynamic power consumption
  - Static power consumption

Dynamic Power Consumption

- Power to charge transistor gate capacitances
  - Energy required to charge a capacitance, C, to V_{DD} is CV_{DD}^2
  - Circuit running at frequency f: transistors switch (from 1 to 0 or vice versa) at that frequency
  - Capacitor is charged $f/2$ times per second (discharging from 1 to 0 is free)
- Dynamic power consumption

$$P_{dynamic} = \frac{1}{2} C V_{DD}^2 f$$

Static Power Consumption

- Power consumed when no gates are switching
- Caused by the quiescent supply current, I_{DD} (also called the leakage current)

- Static power consumption

$$P_{static} = I_{DD}V_{DD}$$

Power Consumption Example

- Estimate the power consumption of a wireless handheld computer
  - $V\_DD = 1.2$ V
  - $C = 20$ nF
  - $f = 1$ GHz
  - $I_{DD} = 20$ mA
- Total power is sum of dynamic and static

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Power Consumption Example

- Estimate the power consumption of a wireless handheld computer
  - $V_DD = $ 1.2 V
  - $C = $ 20 nF
  - $f = $ 1 GHz
  - $I_{DD} = $ 20 mA
- Total power is sum of dynamic and static

$$P = \frac{1}{2} C V_{DD}^2 f + I_{DD} V_{DD}$$
$$= \frac{1}{2} (20\text{ n})(1.2)^2 (1\text{ G})$$
$$+ (20\text{ m})(1.2)$$
$$= (14.4 + 0.024)\text{W}$$
$$= 14.4\text{ W}$$

Extras

Binary Codes

Another way of representing decimal numbers

Example binary codes:

- Weighted codes
  - Binary Coded Decimal (BCD) (8-4-2-1 code)
  - 6-3-1-1 code
  - 8-4-2-1 code (simple binary)
- Gray codes
- Excess-3 code
- 2-out-of-5 code

Binary Codes

Decimal #	8-4-2-1 (BCD)	6-3-1-1	Excess-3	2-out-of-5	Gray
0	0000	0000	0011	00011	0000
1	0001	0001	0100	00101	0001
2	0010	0011	0101	00110	0011
3	0011	0100	0110	01001	0010
4	0100	0101	0111	01010	0110
5	0101	0111	1000	01100	1110
6	0110	1000	1001	10001	1010
7	0111	1001	1010	10010	1011
8	1000	1011	1011	10100	1001
9	1001	1100	1100	11000	1000

Each code combination represents **a single decimal digit.**

ASCII-Code

TABLE 1-3 ASCII Code

	ASCII Code								ASCII Code								ASCII Code						
Character	A_6	A_5	A_4	A_3	A_2	A_1	A_0	Character	A_6	A_5	A_4	A_3	A_2	A_1	A_0	Character	A_6	A_5	A_4	A_3	A_2	A_1	A_0
space	0	1	0	0	0	0	0	@	1	0	0	0	0	0	0	‘	1	1	0	0	0	0	0
!	0	1	0	0	0	0	1	A	1	0	0	0	0	0	1	a	1	1	0	0	0	0	1
"	0	1	0	0	0	1	0	B	1	0	0	0	0	1	0	b	1	1	0	0	0	1	0
#	0	1	0	0	0	1	1	C	1	0	0	0	0	1	1	c	1	1	0	0	0	1	1
$	0	1	0	0	1	0	0	D	1	0	0	0	1	0	0	d	1	1	0	0	1	0	0
%	0	1	0	0	1	0	1	E	1	0	0	0	1	0	1	e	1	1	0	0	1	0	1
&	0	1	0	0	1	1	0	F	1	0	0	0	1	1	0	f	1	1	0	0	1	1	0
'	0	1	0	0	1	1	1	G	1	0	0	0	1	1	1	g	1	1	0	0	1	1	1
(	0	1	0	1	0	0	0	H	1	0	0	1	0	0	0	h	1	1	0	1	0	0	0
)	0	1	0	1	0	0	1	I	1	0	0	1	0	0	1	i	1	1	0	1	0	0	1
*	0	1	0	1	0	1	0	J	1	0	0	1	0	1	0	j	1	1	0	1	0	1	0
+	0	1	0	1	0	1	1	K	1	0	0	1	0	1	1	k	1	1	0	1	0	1	1
,	0	1	0	1	1	0	0	L	1	0	0	1	1	0	0	l	1	1	0	1	1	0	0
–	0	1	0	1	1	0	1	M	1	0	0	1	1	0	1	m	1	1	0	1	1	0	1
.	0	1	0	1	1	1	0	N	1	0	0	1	1	1	0	n	1	1	0	1	1	1	0
/	0	1	0	1	1	1	1	O	1	0	0	1	1	1	1	o	1	1	0	1	1	1	1
0	0	1	1	0	0	0	0	P	1	0	1	0	0	0	0	p	1	1	1	0	0	0	0
1	0	1	1	0	0	0	1	Q	1	0	1	0	0	0	1	q	1	1	1	0	0	0	1
2	0	1	1	0	0	1	0	R	1	0	1	0	0	1	0	r	1	1	1	0	0	1	0
3	0	1	1	0	0	1	1	S	1	0	1	0	0	1	1	s	1	1	1	0	0	1	1
4	0	1	1	0	1	0	0	T	1	0	1	0	1	0	0	t	1	1	1	0	1	0	0
5	0	1	1	0	1	0	1	U	1	0	1	0	1	0	1	u	1	1	1	0	1	0	1
6	0	1	1	0	1	1	0	V	1	0	1	0	1	1	0	v	1	1	1	0	1	1	0
7	0	1	1	0	1	1	1	W	1	0	1	0	1	1	1	w	1	1	1	0	1	1	1
8	0	1	1	1	0	0	0	X	1	0	1	1	0	0	0	x	1	1	1	1	0	0	0
9	0	1	1	1	0	0	1	Y	1	0	1	1	0	0	1	y	1	1	1	1	0	0	1
:	0	1	1	1	0	1	0	Z	1	0	1	1	0	1	0	z	1	1	1	1	0	1	0
;	0	1	1	1	0	1	1	[	1	0	1	1	0	1	1	{	1	1	1	1	0	1	1
<	0	1	1	1	1	0	0	\	1	0	1	1	1	0	0	|	1	1	1	1	1	0	0
=	0	1	1	1	1	0	1	]	1	0	1	1	1	0	1	}	1	1	1	1	1	0	1
>	0	1	1	1	1	1	0	^	1	0	1	1	1	1	0	~	1	1	1	1	1	1	0
?	0	1	1	1	1	1	1	—	1	0	1	1	1	1	1	delete	1	1	1	1	1	1	1

Weighted Codes

- Weighted codes: each bit position has a given weight
  - Binary Coded Decimal (BCD) (8-4-2-1 code)
    - **Example:** **726_{10}** = **0111 0010 0110**$_{BCD}$
  - 6-3-1-1 code
    - **Example:** 1001 (6-3-1-1 code) = 1×6 + 0×3 + 0×1 + 1×1
    - **Example:** **726_{10}** = **1001 0011 1000**$_{6311}$
- BCD numbers are used to represent fractional numbers exactly (vs. floating point numbers – which can't - see Chapter 5)

Weighted Codes

Decimal #	8-4-2-1 (BCD)	6-3-1-1
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0100
4	0100	0101
5	0101	0111
6	0110	1000
7	0111	1001
8	1000	1011
9	1001	1100

- **BCD Example:**

726_{10} = **0111 0010 0110**$_{BCD}$

- **6-3-1-1 code Example:**

726_{10} = **1001 0011 1000**$_{6311}$

Excess-3 Code

Decimal #	Excess-3
0	0011
1	0100
2	0101
3	0110
4	0111
5	1000
6	1001
7	1010
8	1011
9	1100

- Add 3 to number, then represent in binary
  - **Example:** 5_{10} = 5+3 = 8 = $\mathbf{1000_2}$
- Also called a **biased** number
- Excess-3 codes (also called XS-3) were used in the 1970's to ease arithmetic

- **Excess-3 Example:**

726_{10} = **1010 0101 1001**$_{xs3}$

2-out-of-5 Code

Decimal #	2-out-of-5
0	00011
1	00101
2	00110
3	01001
4	01010
5	01100
6	10001
7	10010
8	10100
9	11000

- 2 out of the 5 bits are 1
- Used for error detection:
  - If more or less than 2 of 5 bits are 1, error

Gray Codes

Decimal #	Gray
0	0000
1	0001
2	0011
3	0010
4	0110
5	1110
6	1010
7	1011
8	1001
9	1000

- Next number differs in only one bit position
  - **Example:** 000, 001, 011, 010, 110, 111, 101, 100
- **Example use:** Analog-to-Digital (A/D) converters. Changing 2 bits at a time (i.e., 011 →100) could cause large inaccuracies.