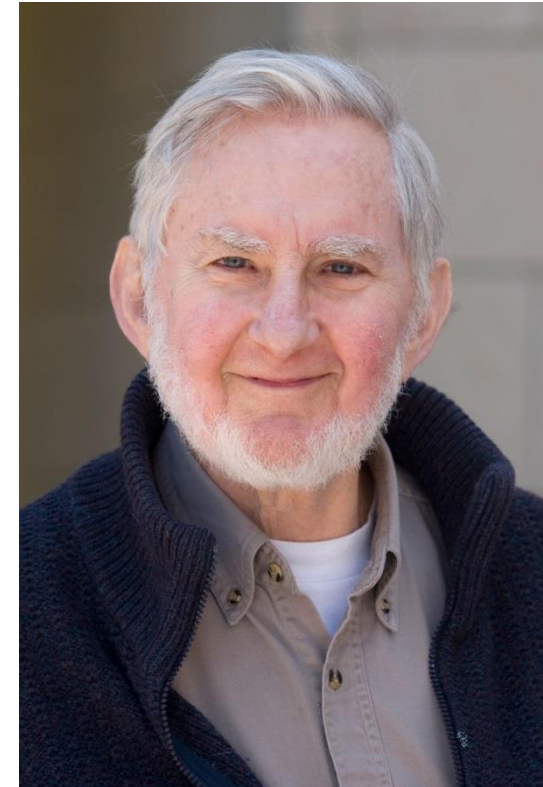


Quine-McCluskey Algorithm

- Useful for minimizing equations with more than 4 inputs.
- Like K-map, also uses **combining** theorem
- Allows for automation

Edward McCluskey (1929-2016)

- Pioneer in Electrical Engineering
- First president of IEEE
- Professor at Princeton, then Stanford
- 1955: Quine-McCluskey Algorithm



Quine-McCluskey Algorithm

Method:

1. 1's Table:

- List each minterm, sorted by the number of 1's it contains.
- Combine minterms.

2. Prime Implicant Table: When you can't combine anymore, list prime implicants and the minterms they cover.

3. Select Prime Implicants to cover all minterms.

Quine-McCluskey Example 1

ABC	Y
000	1
001	1
010	1
011	1
100	0
101	1
110	1
111	0

Quine-McCluskey Example 1

<i>ABC</i>	<i>Y</i>
000	1
001	1
010	1
011	1
100	0
101	1
110	1
111	0

1's Table

Number of 1's	Size 1 Implicants	
0	000 m0	
1	001 m1	
	010 m2	
2	011 m3	
	101 m5	
	110 m6	

Order the minterms by the **number of 1's** they have.

Quine-McCluskey Example 1

<i>ABC</i>	<i>Y</i>
000	1
001	1
010	1
011	1
100	0
101	1
110	1
111	0

1's Table

Number of 1's	Size 1 Implicants	Size 2 Implicants	
0	000 m0	00- m(0,1) 0-0 m(0,2)	
1	001 m1 010 m2	0-1 m(1,3) -01 m(1,5) 01- m(2,3) -10 m(2,6)	
2	011 m3 101 m5 110 m6		

Combine minterms in adjacent groups (starting with the top group).

- To combine terms, minterm can **differ** (be 1 or 0) in only 1 place
- '-' indicates "don't care"

Quine-McCluskey Example 1

<i>ABC</i>	<i>Y</i>	1's Table	Number of 1's	Size 1 Implicants	Size 2 Implicants	Size 4 Implicants
000	1		0	000 m0	00- m(0,1) 0-0 m(0,2)	0-- m(0,1,2,3)
001	1		1	001 m1	0-1 m(1,3) -01 m(1,5)	
010	1		2	010 m2	01- m(2,3) -10 m(2,6)	
011	1					
100	0					
101	1					
110	1					
111	0					

Combine minterms in adjacent groups (starting with the top group).

- To combine terms, minterm can **differ** (be 1 or 0) in only 1 place
- '-' (don't care) acts like another **variable** (0, 1, -)
- Match up '-' **first**.

Quine-McCluskey Example 1

<i>ABC</i>	<i>Y</i>	1's Table	Number of 1's	Size 1 Implicants	Size 2 Implicants	Size 4 Implicants
000	1		0	000 m0	00- m(0,1) 0-0 m(0,2)	0-- m(0,1,2,3)*
001	1		1	001 m1	0-1 m(1,3) -01 m(1,5)*	
010	1		2	010 m2	01- m(2,3) -10 m(2,6)*	
011	1			011 m3		
100	0			101 m5		
101	1			110 m6		
110	1					
111	0					

List **prime implicants**:

- **Largest implicants** containing a given minterm
- For example m(0,1) is an implicant **but not a prime implicant**, because m(0,1,2,3) is a larger implicant containing those minterms.
- However, m(1,5) is a prime implicant – no larger implicant exists that contains minterm 5.



Quine-McCluskey Example 1

<i>ABC</i>	<i>Y</i>
000	1
001	1
010	1
011	1
100	0
101	1
110	1
111	0

1's Table

Number of 1's	Size 1 Implicants	Size 2 Implicants	Size 4 Implicants
0	000 m0	00- m(0,1) 0-0 m(0,2)	0-- m(0,1,2,3)*
1	001 m1 010 m2	0-1 m(1,3) -01 m(1,5)* 01- m(2,3) -10 m(2,6)*	
2	011 m3 101 m5 110 m6		

Prime Implicant Table

Prime Implicants	Minterms						ABC
	0	1	2	3	5	6	
m(0,1,2,3)	X	X	X	X			0 - -
m(1,5)		X			X		- 01
m(2,6)			X			X	- 10

List prime implicants.

Show which of the required minterms each includes.

Quine-McCluskey Example 1

ABC	Y
000	1
001	1
010	1
011	1
100	0
101	1
110	1
111	0

1's Table

Number of 1's	Size 1 Implicants	Size 2 Implicants	Size 4 Implicants
0	000 m0	00- m(0,1) 0-0 m(0,2)	0-- m(0,1,2,3)*
1	001 m1 010 m2	0-1 m(1,3) -01 m(1,5)* 01- m(2,3) -10 m(2,6)*	
2	011 m3 101 m5 110 m6		

Prime Implicant Table

Prime Implicants	Minterms						ABC
	0	1	2	3	5	6	
m(0,1,2,3)	X	X	X	X			0 - -
m(1,5)		X			X		- 01
m(2,6)			X			X	- 10

Select columns with **only 1 X**.

Corresponding prime implicants must be included in equation.

Quine-McCluskey Example 1

<i>ABC</i>	<i>Y</i>
000	1
001	1
010	1
011	1
100	0
101	1
110	1
111	0

1's Table

Number of 1's	Size 1 Implicants	Size 2 Implicants	Size 4 Implicants
0	000 m0	00- m(0,1) 0-0 m(0,2)	0-- m(0,1,2,3)*
1	001 m1 010 m2	0-1 m(1,3) -01 m(1,5)* 01- m(2,3) -10 m(2,6)*	
2	011 m3 101 m5 110 m6		

Prime Implicant Table

Prime Implicants	Minterms						ABC
	0	1	2	3	5	6	
m(0,1,2,3)	X	X	X	X			0 - -
m(1,5)		X			X		- 01
m(2,6)			X			X	- 10

Select columns with **only 1 X**.

Corresponding prime implicants must be included in equation.

Quine-McCluskey Example 1

<i>ABC</i>	<i>Y</i>
000	1
001	1
010	1
011	1
100	0
101	1
110	1
111	0

1's Table

Number of 1's	Size 1 Implicants	Size 2 Implicants	Size 4 Implicants
0	000 m0	00- m(0,1) 0-0 m(0,2)	0-- m(0,1,2,3)*
1	001 m1 010 m2	0-1 m(1,3) -01 m(1,5)* 01- m(2,3) -10 m(2,6)*	
2	011 m3 101 m5 110 m6		

Prime Implicant Table

Prime Implicants	Minterms						ABC
	0	1	2	3	5	6	
m(0,1,2,3)	X	X	X	X			0 - -
m(1,5)		X			X		- 01
m(2,6)			X			X	- 10

$$Y = \overline{A} + \overline{B}C + B\overline{C}$$

Select columns with **only 1 X**.

Corresponding prime implicants must be included in equation.

Quine-McCluskey Example 2

ABCD	Y
0000	0
0001	X
0010	1
0011	X
0100	1
0101	0
0110	X
0111	0
1000	1
1001	1
1010	X
1011	0
1100	0
1101	0
1110	1
1111	0

Quine-McCluskey Example 2

1's Table

<i>ABCD</i>	<i>Y</i>
0000	0
0001	X
0010	1
0011	X
0100	1
0101	0
0110	X
0111	0
1000	1
1001	1
1010	X
1011	0
1100	0
1101	0
1110	1
1111	0

Number of 1's	Size 1 Implicants	
1	0001 m1 0010 m2 0100 m4 1000 m8	
2	0011 m3 0110 m6 1001 m9 1010 m10	
3	1110 m14	

Quine-McCluskey Example 2

1's Table

ABCD	Y
0000	0
0001	X
0010	1
0011	X
0100	1
0101	0
0110	X
0111	0
1000	1
1001	1
1010	X
1011	0
1100	0
1101	0
1110	1
1111	0

Number of 1's	Size 1 Implicants	Size 2 Implicants	
1	0001 m1 0010 m2 0100 m4 1000 m8	00-1 m(1,3) -001 m(1,9) 001- m(2,3) 0-10 m(2,6) -010 m(2,10) 01-0 m(4,6) 100- m(8,9) 10-0 m(8,10)	
2	0011 m3 0110 m6 1001 m9 1010 m10	-110 m(6,14) 1-10 m(10,14)	
3	1110 m14		

Quine-McCluskey Example 2

1's Table

ABCD	Y
0000	0
0001	X
0010	1
0011	X
0100	1
0101	0
0110	X
0111	0
1000	1
1001	1
1010	X
1011	0
1100	0
1101	0
1110	1
1111	0

Number of 1's	Size 1 Implicants	Size 2 Implicants	Size 4 Implicants
1	0001 m1 0010 m2 0100 m4 1000 m8	00-1 m(1,3) -001 m(1,9) 001- m(2,3) 0-10 m(2,6) -010 m(2,10) 01-0 m(4,6) 100- m(8,9) 10-0 m(8,10)	--10 m(2,6,10,14)
2	0011 m3 0110 m6 1001 m9 1010 m10	-110 m(6,14) 1-10 m(10,14)	
3	1110 m14		

Quine-McCluskey Example 2

1's Table

ABCD	Y
0000	0
0001	X
0010	1
0011	X
0100	1
0101	0
0110	X
0111	0
1000	1
1001	1
1010	X
1011	0
1100	0
1101	0
1110	1
1111	0

Number of 1's	Size 1 Implicants	Size 2 Implicants	Size 4 Implicants
1	0001 m1 0010 m2 0100 m4 1000 m8	00-1 m(1,3)* -001 m(1,9)* 001- m(2,3)* 0-10 m(2,6) -010 m(2,10) 01-0 m(4,6)* 100- m(8,9)* 10-0 m(8,10)*	--10 m(2,6,10,14)*
2	0011 m3 0110 m6 1001 m9 1010 m10	-110 m(6,14) 1-10 m(10,14)	
3	1110 m14		

Quine-McCluskey Example 2

ABCD	Y
0000	0
0001	X
0010	1
0011	X
0100	1
0101	0
0110	X
0111	0
1000	1
1001	1
1010	X
1011	0
1100	0
1101	0
1110	1
1111	0

1's Table

Prime Implicant Table

Number of 1's	Size 1 Implicants	Size 2 Implicants	Size 4 Implicants
1	0001 m1 0010 m2 0100 m4 1000 m8	00-1 m(1,3)* -001 m(1,9)* 001- m(2,3)* 0-10 m(2,6) -010 m(2,10) 01-0 m(4,6)* 100- m(8,9)* 10-0 m(8,10)*	--10 m(2,6,10,14)*
2	0011 m3 0110 m6 1001 m9 1010 m10	-110 m(6,14) 1-10 m(10,14)	
3	1110 m14		

Prime Implicants	Minterms					ABCD
	2	4	8	9	14	
m(2,6,10,14)	X				X	--10
m(1,3)						00-1
m(1,9)				X		-001
m(2,3)	X					001-
m(4,6)		X				01-0
m(8,9)			X	X		100-
m(8,10)			X			10-0

Quine-McCluskey Example 2

<i>ABCD</i>	<i>Y</i>
0000	0
0001	X
0010	1
0011	X
0100	1
0101	0
0110	X
0111	0
1000	1
1001	1
1010	X
1011	0
1100	0
1101	0
1110	1
1111	0

Prime Implicant
Table

Prime Implicants	Minterms					ABCD
	2	4	8	9	14	
m(2,6,10,14)	X				X	--10
m(1,3)						00-1
m(1,9)				X		-001
m(2,3)	X					001-
m(4,6)		X				01-0
m(8,9)			X	X		100-
m(8,10)			X			10-0

Quine-McCluskey Example 2

ABCD	Y
0000	0
0001	X
0010	1
0011	X
0100	1
0101	0
0110	X
0111	0
1000	1
1001	1
1010	X
1011	0
1100	0
1101	0
1110	1
1111	0

Prime Implicant
Table

Prime Implicants	Minterms					ABCD
	2	4	8	9	14	
m(2,6,10,14)	X				X	--10
m(1,3)						00-1
m(1,9)				X		-001
m(2,3)	X					001-
m(4,6)		X				01-0
m(8,9)			X	X		100-
m(8,10)			X			10-0

Quine-McCluskey Example 2

ABCD	Y
0000	0
0001	X
0010	1
0011	X
0100	1
0101	0
0110	X
0111	0
1000	1
1001	1
1010	X
1011	0
1100	0
1101	0
1110	1
1111	0

Prime Implicant
Table

Prime Implicants	Minterms					ABCD
	2	4	8	9	14	
m(2,6,10,14)	X				X	--10
m(1,3)						00-1
m(1,9)				X		-001
m(2,3)	X					001-
m(4,6)		X				01-0
m(8,9)			X	X		100-
m(8,10)			X			10-0

Quine-McCluskey Example 2

ABCD	Y
0000	0
0001	X
0010	1
0011	X
0100	1
0101	0
0110	X
0111	0
1000	1
1001	1
1010	X
1011	0
1100	0
1101	0
1110	1
1111	0

Prime Implicant
Table

Prime Implicants	Minterms					ABCD
	2	4	8	9	14	
m(2,6,10,14)	X				X	--10
m(1,3)						00-1
m(1,9)				X		-001
m(2,3)	X					001-
m(4,6)		X				01-0
m(8,9)			X	X		100-
m(8,10)			X			10-0

Quine-McCluskey Example 2

ABCD	Y
0000	0
0001	X
0010	1
0011	X
0100	1
0101	0
0110	X
0111	0
1000	1
1001	1
1010	X
1011	0
1100	0
1101	0
1110	1
1111	0

Prime Implicant
Table

Prime Implicants	Minterms					ABCD
	2	4	8	9	14	
m(2,6,10,14)	X				X	--10
m(1,3)						00-1
m(1,9)				X		-001
m(2,3)	X					001-
m(4,6)		X				01-0
m(8,9)			X	X		100-
m(8,10)			X			10-0

$$Y = \overline{C}\overline{D} + \overline{A}\overline{B}\overline{D} + \overline{A}\overline{B}\overline{C}$$