T9: Covering

Number	Theorem	Name
T9	B• (B+C) = B	Covering

Prove true by:

- **Method 1:** Perfect induction
- **Method 2:** Using other theorems and axioms

T9: Covering

Number	Theorem	Name
T9	B• (B+C) = B	Covering

Method 1: Perfect Induction

B	C	(B+C)	B(B+C)
0	0		
0	1		
1	0		
1	1		

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T9: Covering

Number	Theorem	Name
T9	B• (B+C) = B	Covering

Method 1: Perfect Induction

B	*C*	*(B+C)*	*B(B+C)*
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

T9: Covering

Number	Theorem	Name
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Method 2: Prove true using other axioms and theorems.

T9: Covering

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Method 2: Prove true using other axioms and theorems.

B•(B+C)	= B•B + B•C	T8: Distributivity
	= **B** + B•C	T3: Idempotency
	= B•(1 + C)	T8: Distributivity
	= B•(**1**)	T2: Null element
	= B	T1: Identity

T10: Combining

Number	Theorem	Name
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	Combining

Prove true using other axioms and theorems:

T10: Combining

Number	Theorem	Name
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	Combining

Prove true using other axioms and theorems:

$B \bullet C + B \bullet \overline{C} = B \bullet (C + \overline{C})$ T8: Distributivity

$= B \bullet (\mathbf{1})$ T5': Complements

$= B$ T1: Identity

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T11: Consensus

Number	Theorem	Name
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) = (B \bullet C) + \overline{B} \bullet D$	Consensus

Prove true using (1) perfect induction or (2) other axioms and theorems.

Recap: Boolean Thms of Several Vars

Number	Theorem	Name
T6	$B \bullet C = C \bullet B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	Associativity
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	Distributivity
T9	$B \bullet (B+C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	Combining
T11	$B \bullet C + (\overline{B} \bullet D) + (C \bullet D) =$ $B \bullet C + \overline{B} \bullet D$	Consensus

Boolean Thms of Several Vars: Duals

#	Theorem	Dual	Name
T6	B•C = C•B	B+C = C+B	Commutativity
T7	(B•C) • D = B • (C•D)	(B + C) + D = B + (C + D)	Associativity
T8	B • (C + D) = (B•C) + (B•D)	B + (C•D) = (B+C) (B+D)	Distributivity
T9	B • (B+C) = B	B + (B•C) = B	Covering
T10	(B•C) + (B•$\overline{C}$) = B	(B+C) • (B+$\overline{C}$) = B	Combining
T11	(B•C) + (B•$\overline{D}$) + (C•D) = (B•C) + (B•$\overline{D}$)	(B+C) • (B+$\overline{D}$) • (C+D) = (B+C) • (B+$\overline{D}$)	Consensus

Dual: Replace: • with +
0 with 1

Boolean Thms of Several Vars: Duals

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T10	(B•C) + (B•$\overline{C}$) = B	(B+C) • (B+$\overline{C}$) = B	Combining
T11	(B•C) + (B•$\overline{D}$) + (C•D) = (B•C) + (B•$\overline{D}$)	(B+C) • (B+$\overline{D}$) • (C+D) = (B+C) • (B+$\overline{D}$)	Consensus

Dual: Replace: • with +
0 with 1

Warning: T8' differs from traditional algebra: OR (+) distributes over AND (•)

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Boolean Thms of Several Vars: Duals

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Axioms and theorems are useful for *simplifying* equations.

Simplifying an Equation

- Reducing an equation to the fewest number of implicants, where each implicant has the fewest literals

Simplifying an Equation

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Recall:

- **Implicant:** product of literals
  $AB\overline{C}, AC, \overline{B}C$
- **Literal:** variable or its complement
  $A, \overline{A}, B, \overline{B}, C, \overline{C}$

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- **Implicant:** product of literals
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- **Literal:** variable or its complement
  A, $\overline{A}$, B, $\overline{B}$, C, $\overline{C}$

- Also called: **minimizing** the equation

Simplification methods

- **Distributivity (T8, T8')**

  B (C+D) = BC + BD

  B + CD = (B+ C)(B+D)

- **Covering (T9')**

  A + AP = A

- **Combining (T10)**

  $PA + P\overline{A} = P$

Simplification methods

- **Distributivity (T8, T8')**

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- **Combining (T10)**

  $PA + P\overline{A} = P$
- **Expansion**

  $P = PA + P\overline{A}$

  A = A + AP
- **Duplication**

  A = A + A

Simplification methods

- **Distributivity (T8, T8')**

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- **A combination of Combining/Covering**

  $PA + \overline{A} = P + \overline{A}$

Simplification methods

- **A combination of Combining/Covering**

$$P A + \overline{A} = P + \overline{A}$$

Proof: $PA + \overline{A} = PA + (\mathbf{\overline{A} + \overline{A}P})$ **T9' Covering**

$= PA + \mathbf{P\overline{A}} + \overline{A}$ **T6 Commutativity**

$= \mathbf{P(A + \overline{A})} + \overline{A}$ **T8 Distributivity**

$= P\mathbf{(1)} + \overline{A}$ **T5' Complements**

$= P + \overline{A}$ **T1 Identity**

T11: Consensus

Number	Theorem	Name
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) = (B \bullet C) + \overline{B} \bullet D$	Consensus

Prove using other theorems and axioms:

T11: Consensus

Number	Theorem	Name
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) = (B \bullet C) + \overline{B} \bullet D$	Consensus

Prove using other theorems and axioms:

$B \bullet C + \overline{B} \bullet D + C \bullet D$

$= BC + \overline{B}D + (CDB + CD\overline{B})$ **T10: Combining**

$= BC + \overline{B}D + BCD + \overline{B}CD$ **T6: Commutativity**

$= BC + BCD + \overline{B}D + \overline{B}CD$ **T6: Commutativity**

$= (BC + BCD) + (\overline{B}D + \overline{B}CD)$ **T7: Associativity**

$= BC + \overline{B}D$ **T9': Covering**

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Recap: Boolean Thms of Several Vars

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T6	B•C = C•B	B+C = C+B	Commutativity
T7	(B•C) • D = B • (C•D)	(B + C) + D = B + (C + D)	Associativity
T8	B • (C + D) = (B•C) + (B•D)	B + (C•D) = (B+C) (B+D)	Distributivity
T9	B • (B+C) = B	B + (B•C) = B	Covering
T10	(B•C) + (B•$\overline{C}$) = B	(B+C) • (B+$\overline{C}$) = B	Combining
T11	(B•C) + (B•$\overline{D}$) + (C•D) = (B•C) + (B•$\overline{D}$)	(B+C) • (B+$\overline{D}$) • (C+D) = (B+C) • (B+$\overline{D}$)	Consensus

Simplification methods

COMBINATIONAL LOGIC DESIGN

- **Distributivity (T8, T8')**

  $B(C+D) = BC + BD$

  $B + CD = (B+C)(B+D)$

- **Covering (T9')**

  $A + AP = A$

- **Combining (T10)**

  $PA + P\overline{A} = P$

- **Expansion**

  $P = PA + P\overline{A}$

  $A = A + AP$

- **Duplication**

  $A = A + A$

- **A combination of Combining/Covering**

  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 1:

$$Y = AB + \overline{A}B$$

Simplifying Boolean Equations

Example 1:

$Y = AB + \overline{A}B$

$Y = A$	T10: Combining
or	
$= A(B + \overline{B})$	T8: Distributivity
$= A(1)$	T5': Complements
$= A$	T1: Identity

Simplification methods

- **Distributivity (T8, T8')**

  $B(C+D) = BC + BD$

  $B + CD = (B+C)(B+D)$

- **Covering (T9')**

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- **Duplication**

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- **A combination of Combining/Covering**

  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 2:

$Y = A(AB + ABC)$

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Simplifying Boolean Equations

Example 2:

$Y = A(AB + ABC)$

$= A(AB(1 + C))$	T8: Distributivity
$= A(AB(1))$	T2': Null Element
$= A(AB)$	T1: Identity
$= (AA)B$	T7: Associativity
$= AB$	T3: Idempotency

Simplification methods

- **Distributivity (T8, T8')**

  $B(C+D) = BC + BD$

  $B + CD = (B+C)(B+D)$

- **Covering (T9')**

  $A + AP = A$

- **Combining (T10)**

  $PA + P\overline{A} = P$

- **Expansion**

  $P = PA + P\overline{A}$

  $A = A + AP$

- **Duplication**

  $A = A + A$

- **A combination of Combining/Covering**

  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 3:

Y = A'BC + A'

Recall: A' = $\overline{A}$

Simplifying Boolean Equations

Example 3:

Y = A'BC + A' — **Recall:** **A' = $\overline{A}$**

= A' — T9' Covering: X + XY = X

or

= A'(BC + 1) — T8: Distributivity

= A'(1) — T2': Null Element

= A' — T1: Identity

COMBINATIONAL LOGIC DESIGN

Simplification methods

- **Distributivity (T8, T8')**

  $B(C+D) = BC + BD$

  $B + CD = (B+C)(B+D)$

- **Covering (T9')**

  $A + AP = A$

- **Combining (T10)**

  $PA + P\overline{A} = P$

- **Expansion**

  $P = PA + P\overline{A}$

  $A = A + AP$

- **Duplication**

  $A = A + A$

- **A combination of Combining/Covering**

  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 4:

$Y = AB'C + ABC + A'BC$

Simplifying Boolean Equations

Example 4:

Y = AB'C + ABC + A'BC

= AB'C + **ABC + ABC** + A'BC	T3': Idempotency
= (AB'C+ABC) + (ABC+A'BC)	T7': Associativity
= AC + BC	T10: Combining

Simplification methods

- **Distributivity (T8, T8')**

  $B(C+D) = BC + BD$

  $B + CD = (B+C)(B+D)$

- **Covering (T9')**

  $A + AP = A$

- **Combining (T10)**

  $PA + P\overline{A} = P$

- **Expansion**

  $P = PA + P\overline{A}$

  $A = A + AP$

- **Duplication**

  $A = A + A$

- **A combination of Combining/Covering**

  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 5:

$Y = AB + BC + B'D' + AC'D'$

Simplifying Boolean Equations

Example 5:

$Y = AB + BC + B'D' + AC'D'$

Method 1:

$Y = AB + BC + B'D' + (ABC'D' + AB'C'D')$	T10: Combining
$= (AB + ABC'D') + BC + (B'D' + AB'C'D')$	T6: Commutativity
	T7: Associativity
$= AB + BC + B'D'$	T9: Covering

Method 2:

$Y = \mathbf{AB} + BC + \mathbf{B'D'} + AC'D' + \mathbf{AD'}$	T11: Consensus
$= AB + BC + B'D' + \mathbf{AD'}$	T9: Covering
$= AB + BC + B'D'$	T11: Consensus

COMBINATIONAL LOGIC DESIGN

Simplification methods

- **Distributivity (T8, T8')**

  $B(C+D) = BC + BD$

  $B + CD = (B+C)(B+D)$

- **Covering (T9')**

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  $PA + P\overline{A} = P$

- **Expansion**

  $P = PA + P\overline{A}$

  $A = A + AP$

- **Duplication**

  $A = A + A$

- **A combination of Combining/Covering**

  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 6:

$Y = (A + BC)(A + DE)$

Apply T8' first when possible: W+XZ = (W+X)(W+Z)

Simplifying Boolean Equations

Example 6:

Y = (A + BC)(A + DE)

Apply T8' first when possible: W+XZ = (W+X)(W+Z)

Make: X = BC, Z = DE and rewrite equation

Y	= (A+X)(A+Z)	substitution (X=BC, Z=DE)
	= A + XZ	T8': Distributivity
	= A + BCDE	substitution

or

Y	= AA + ADE + ABC + BCDE	T8: Distributivity
	= A + ADE + ABC + BCDE	T3: Idempotency
	= **A + ADE** + ABC + BCDE	
	= **A** + ABC + BCDE	T9': Covering
	= A + BCDE	T9': Covering

Simplifying Boolean Equations

Example 6:

$Y = (A + BC)(A + DE)$

Apply T8' first when possible: $W+XZ = (W+X)(W+Z)$

Make: $X = BC$, $Z = DE$ and rewrite equation

Y	$= (A+X)(A+Z)$	substitution ($X=BC$, $Z=DE$)
	$= A + XZ$	T8': Distributivity
	$= A + BCDE$	substitution

or

Y	$= AA + ADE + ABC + BCDE$	T8: Distributivity
	$= A + ADE + ABC + BCDE$	T3: Idempotency
	$=$ **$A + ADE$** $+ ABC + BCDE$	
	$=$ **A** $+ ABC + BCDE$	T9': Covering
	$= A + BCDE$	T9': Covering

This is called *multiplying out* an expression to get sum-of-products (SOP) form.

Multiplying Out: SOP Form

An expression is in simplified **sum-of-products (SOP)** form when all products contain literals only.

- SOP form: Y = AB + BC' + DE
- **NOT** SOP form: Y = DF + E(A'+B)
- SOP form: Z = A + BC + DE'F

Multiplying Out: SOP Form

Example:

$$Y = (A + C + D + E)(A + B)$$

Apply T8' first when possible: W+XZ = (W+X)(W+Z)

Multiplying Out: SOP Form

Example:

Y = (A + C + D + E)(A + B)

Apply T8' first when possible: W+XZ = (W+X)(W+Z)

Make: X = (C+D+E), Z = B and rewrite equation

Y	= (A+X)(A+Z)	substitution (X=(C+D+E), Z=B)
	= A + XZ	T8': Distributivity
	= A + (C+D+E)B	substitution
	= A + BC + BD + BE	T8: Distributivity

or

Y	= AA+AB+AC+BC+AD+BD+AE+BE	T8: Distributivity
	= **A**+AB+AC+AD+AE+BC+BD+BE	T3: Idempotency
	= **A** + BC + BD + BE	T9': Covering

Canonical SOP & POS Form

- SOP – sum-of-products

O	C	E	minterm
0	0	0	$\overline{O}\ \overline{C}$
0	1	0	$\overline{O}\ C$
1	0	1	$O\ \overline{C}$
1	1	0	$O\ C$

$$E = O\overline{C}$$
$$= \Sigma(m_2)$$

- POS – product-of-sums

O	C	E	maxterm
0	0	0	$O + C$
0	1	0	$O + \overline{C}$
1	0	1	$\overline{O} + C$
1	1	0	$\overline{O} + \overline{C}$

$$E = (O + C)(O + \overline{C})(\overline{O} + \overline{C})$$
$$= \Pi(M0, M1, M3)$$

Factoring: POS Form

An expression is in simplified **product-of-sums (POS)** form when all sums contain literals only.

- POS form: Y = (A+B)(C+D)(E'+F)
- **NOT** POS form: Y = (D+E)(F'+GH)
- POS form: Z = A(B+C)(D+E')

Factoring: POS Form

Example 1:

Y = (A + B'CDE)

Apply T8' first when possible: W+XZ = (W+X)(W+Z)

Factoring: POS Form

Example 1:

Y = (A + B'CDE)

Apply T8' first when possible: W+XZ = (W+X)(W+Z)

Make: X = B'C, Z = DE and rewrite equation

Y	= (A+XZ)		substitution (X=B'C, Z=DE)
	= (A+B'C)(A+DE)		T8': Distributivity
	= (A+B')(A+C)(A+D)(A+E)		T8': Distributivity

Factoring: POS Form

Example 2:

Y = AB + C'DE + F

Apply T8' first when possible: W+XZ = (W+X)(W+Z)

Factoring: POS Form

Example 2:

Y = AB + C'DE + F

Apply T8' first when possible: W+XZ = (W+X)(W+Z)

Make: W = AB, X = C', Z = DE and rewrite equation

Y	= (W+XZ) + F	substitution W = AB, X = C', Z = DE
	= (W+X)(W+Z) + F	T8': Distributivity
	= (AB+C')(AB+DE)+F	substitution
	= (A+C')(B+C')(AB+D)(AB+E)+F	T8': Distributivity
	= (A+C')(B+C')(A+D)(B+D)(A+E)(B+E)+F	T8': Distributivity
	= (A+C'+F)(B+C'+F)(A+D+F)(B+D+F)(A+E+F)(B+E+F)	T8': Distributivity

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Simplification methods

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