Chapter 2

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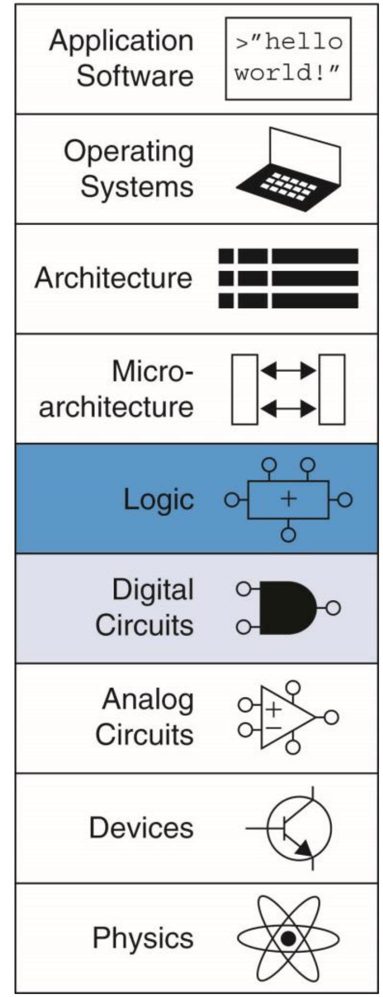
CPE100: Digital Logic Design I

Section 1004: Dr. Morris

Combinational Logic Design

Chapter 2 :: Topics

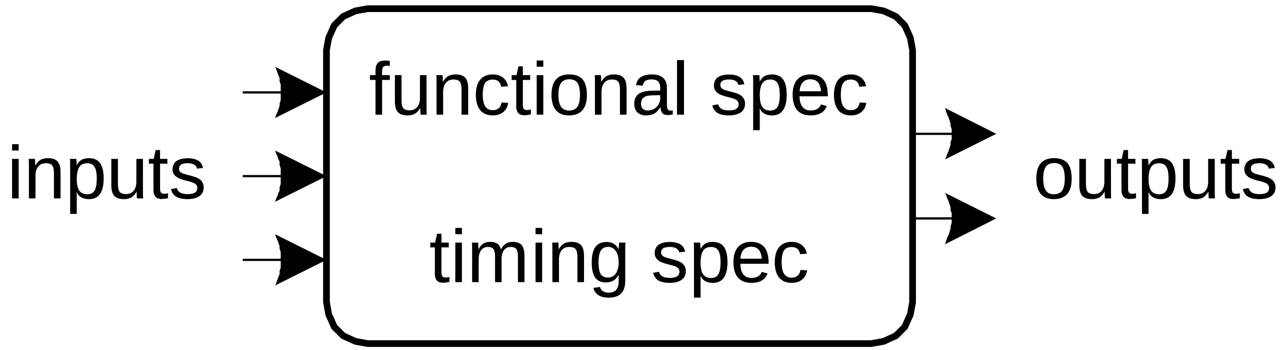
- Introduction
- Boolean Equations
- Boolean Algebra
- From Logic to Gates
- Multilevel Combinational Logic
- X's and Z's, Oh My
- Karnaugh Maps
- Combinational Building Blocks
- Timing



Introduction

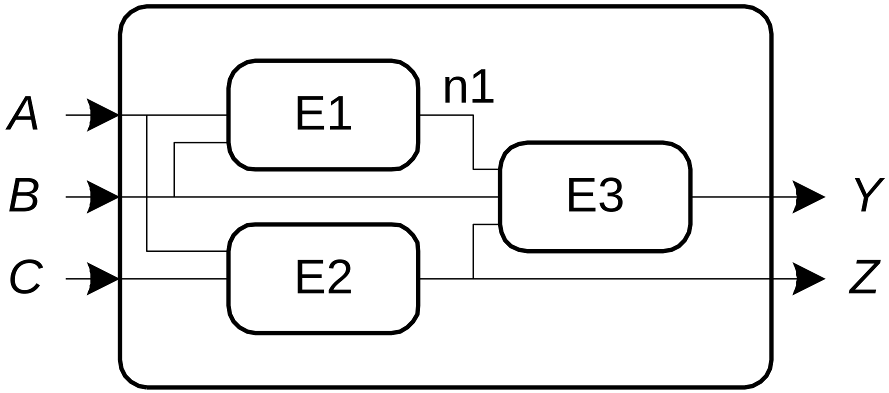
A logic circuit is composed of:

- Inputs
- Outputs
- Functional specification
- Timing specification



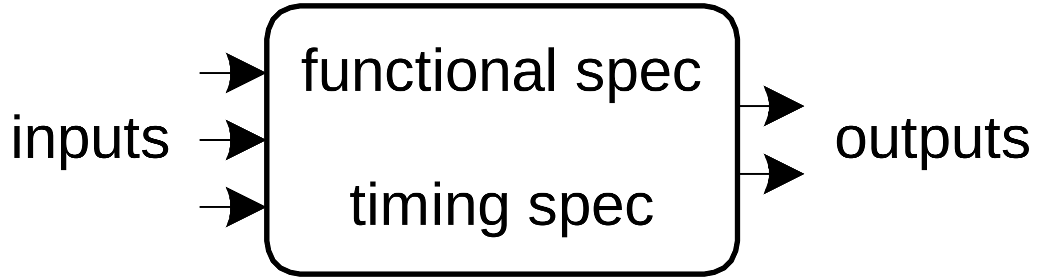
Circuits

- Nodes
  - Inputs: *A*, *B*, *C*
  - Outputs: *Y*, *Z*
  - Internal: n1
- Circuit elements
  - E1, E2, E3
  - Each a circuit



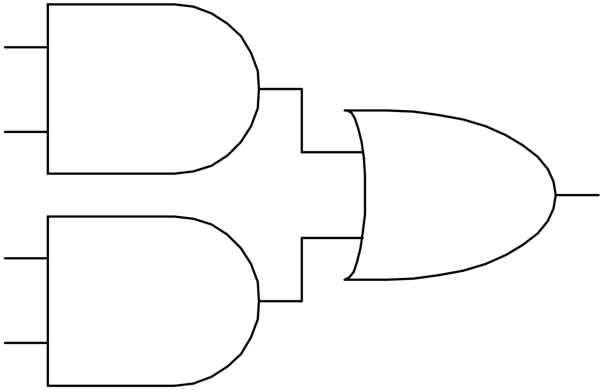
Types of Logic Circuits

- Combinational Logic (Ch 2)
  - Memoryless
  - Outputs determined by current values of inputs
- Sequential Logic (Ch 3)
  - Has memory
  - Outputs determined by previous and current values of inputs



Rules of Combinational Composition

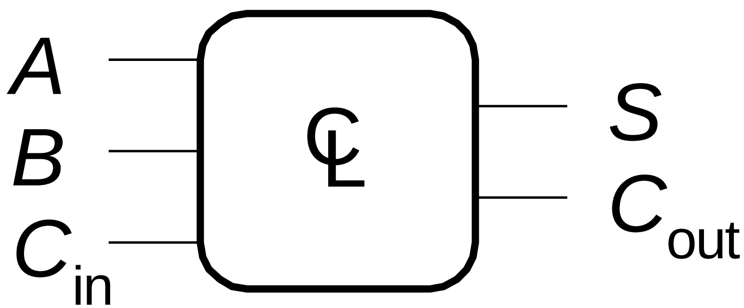
- Every element is combinational
- Every node is either an input or connects to *exactly one* output
- The circuit contains no cyclic paths
  - E.g. no connection from output to internal node
- Example:



Boolean Equations

- Functional specification of outputs in terms of inputs
- **Example:** $S = F(A, B, C_{in})$

  $C_{out} = F(A, B, C_{in})$



$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = AB + AC_{in} + BC_{in}$$

A	B	C_{in}	S	C_{out}

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Functional specification

Goals:

- Systematically express logical functions using Boolean equations
- To simplify Boolean equations

Some Definitions

- Complement: variable with a bar over it

  $\bar{A}, \bar{B}, \bar{C}$

- Literal: variable or its complement

  $A, \bar{A}, B, \bar{B}, C, \bar{C}$

- Implicant: product (AND) of literals

  $AB\bar{C}, AC, BC$

- Minterm: product that includes all input variables

  $AB\bar{C}, A\bar{B}\bar{C}, ABC$

- Maxterm: sum (OR) that includes all input variables

  $(A+B+C), (A+B+\bar{C}), (A+B+C)$

Canonical Sum-of-Products (SOP) Form

- All equations can be written in SOP form
- Each row has a **minterm**
- A minterm is a product (AND) of literals
- Each minterm is TRUE for that row (and only that row)

A	*B*	*Y*	minterm	minterm name
0	0	0	$\overline{A}\ \overline{B}$	m_0
0	1	1	$\overline{A}\ B$	m_1
1	0	0	$A\ \overline{B}$	m_2
1	1	1	$A\ B$	m_3

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- Form function by ORing minterms where the output is TRUE

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Y = F(A, B) =

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- Each row has a **minterm**
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- Each minterm is TRUE for that row (and only that row)
- Form function by ORing minterms where the output is TRUE
- Thus, a sum (OR) of products (AND terms)

A	*B*	*Y*	minterm	minterm name
0	0	0	$\overline{A}\ \overline{B}$	m_0
0	1	1	$\overline{A}\ B$	m_1
1	0	0	$A\ \overline{B}$	m_2
1	1	1	$A\ B$	m_3

$$Y = F(A, B) = \overline{A}B + AB = \Sigma(m_1, m_3)$$

SOP Example

- Steps:
- Find minterms that result in Y=1
- Sum “TRUE” minterms

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

$$Y = \mathbf{F}(A, B) =$$

Aside: Precedence

- AND has precedence over OR
- In other words:
  - AND is performed **before** OR
- Example:
  - $Y = \bar{A} \cdot B + A \cdot B$
  - Equivalent to:
  - $Y = (\bar{A}B) + (AB)$

Canonical Product-of-Sums (POS) Form

- All Boolean equations can be written in POS form
- Each row has a **maxterm**
- A maxterm is a sum (OR) of literals
- Each maxterm is FALSE for that row (and only that row)

A	*B*	*Y*	maxterm	maxterm name
0	0	0	$A + B$	M_0
0	1	1	$A + \overline{B}$	M_1
1	0	0	$\overline{A} + B$	M_2
1	1	1	$\overline{A} + \overline{B}$	M_3

Canonical Product-of-Sums (POS) Form

- All Boolean equations can be written in POS form
- Each row has a **maxterm**
- A maxterm is a sum (OR) of literals
- Each maxterm is FALSE for that row (and only that row)
- Form function by ANDing the maxterms for which the output is FALSE
- Thus, a product (AND) of sums (OR terms)

A	B	Y	maxterm	maxterm name
0	0	0	$A + B$	M_0
0	1	1	$A + \overline{B}$	M_1
1	0	0	$\overline{A} + B$	M_2
1	1	1	$\overline{A} + \overline{B}$	M_3

$$Y = M_0 \cdot M_2 = (A + B) \cdot (\bar{A} + B)$$

SOP and POS Comparison

- Sum of Products (SOP)
  - Implement the “ones” of the output
  - Sum all “one” terms → OR results in “one”
- Product of Sums (POS)
  - Implement the “zeros” of the output
  - Multiply “zero” terms → AND results in “zero”

Boolean Equations Example

- You are going to the cafeteria for lunch
  - You will eat lunch (E=1)
  - If it's open (O=1) **and**
  - If they're not serving corndogs (C=0)
- Write a truth table for determining if you will eat lunch (E).

O	*C*	*E*
0	0	
0	1	
1	0	
1	1	

Boolean Equations Example

- You are going to the cafeteria for lunch
  - You will eat lunch (E=1)
  - If it's open (O=1) **and**
  - If they're not serving corndogs (C=0)
- Write a truth table for determining if you will eat lunch (E).

O	*C*	*E*
0	0	0
0	1	0
1	0	1
1	1	0

SOP & POS Form

- SOP – sum-of-products

O	*C*	*E*	minterm
0	0		$\overline{O}\ \overline{C}$
0	1		$\overline{O}\ C$
1	0		$O\ \overline{C}$
1	1		$O\ C$

- POS – product-of-sums

O	*C*	*E*	maxterm
0	0		$O + C$
0	1		$O + \overline{C}$
1	0		$\overline{O} + C$
1	1		$\overline{O} + \overline{C}$

SOP & POS Form

- SOP – sum-of-products

O	C	E	minterm
0	0	0	$\overline{O}\ \overline{C}$
0	1	0	$\overline{O}\ C$
1	0	1	$O\ \overline{C}$
1	1	0	$O\ C$

$$E = O\overline{C}$$
$$= \Sigma(m_2)$$

- POS – product-of-sums

O	C	E	maxterm
0	0	0	$O + C$
0	1	0	$O + \overline{C}$
1	0	1	$\overline{O} + C$
1	1	0	$\overline{O} + \overline{C}$

$$E = (O + C)(O + \overline{C})(\overline{O} + \overline{C})$$
$$= \Pi(M0, M1, M3)$$

SOP & POS Form

- SOP – sum-of-products

O	C	E	minterm
0	0	0	$\overline{O}\ \overline{C}$
0	1	0	$\overline{O}\ C$
1	0	1	$O\ \overline{C}$
1	1	0	$O\ C$

$$E = O\overline{C}$$
$$= \Sigma(m_2)$$

- POS – product-of-sums

O	C	E	maxterm
0	0	0	$O + C$
0	1	0	$O + \overline{C}$
1	0	1	$\overline{O} + C$
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SOP & POS Form

- SOP – sum-of-products

O	C	E	minterm
0	0	0	$\overline{O}\ \overline{C}$
0	1	0	$\overline{O}\ C$
1	0	1	$O\ \overline{C}$
1	1	0	$O\ C$

$$E = O\overline{C}$$
$$= \Sigma(m_2)$$

- POS – product-of-sums

O	C	E	maxterm
0	0	0	$O + C$
0	1	0	$O + \overline{C}$
1	0	1	$\overline{O} + C$
1	1	0	$\overline{O} + \overline{C}$

$$E = (O + C)(O + \overline{C})(\overline{O} + \overline{C})$$
$$= \Pi(M0, M1, M3)$$

Boolean Algebra

- Axioms and theorems to **simplify** Boolean equations
- Like regular algebra, but simpler: variables have only two values (1 or 0)
- **Duality** in axioms and theorems:
  - ANDs and ORs, 0's and 1's interchanged

Boolean Axioms

	Axiom
A1	$B = 0 \text{ if } B \neq 1$
A2	$\overline{0} = 1$
A3	$0 \bullet 0 = 0$
A4	$1 \bullet 1 = 1$
A5	$0 \bullet 1 = 1 \bullet 0 = 0$

Duality

Duality in Boolean axioms and theorems:

- ANDs and ORs, 0's and 1's interchanged

Boolean Axioms

	Axiom
A1	$B = 0$ if $B \neq 1$
A2	$\overline{0} = 1$
A3	$0 \bullet 0 = 0$
A4	$1 \bullet 1 = 1$
A5	$0 \bullet 1 = 1 \bullet 0 = 0$

Boolean Axioms

	Axiom		Dual
A1	$B = 0$ if $B \neq 1$	A1′	$B = 1$ if $B \neq 0$
A2	$\overline{0} = 1$	A2′	$\overline{1} = 0$
A3	$0 \bullet 0 = 0$	A3′	$1 + 1 = 1$
A4	$1 \bullet 1 = 1$	A4′	$0 + 0 = 0$
A5	$0 \bullet 1 = 1 \bullet 0 = 0$	A5′	$1 + 0 = 0 + 1 = 1$

Dual: Exchange: • and +
0 and 1

Boolean Axioms

	Axiom		Dual	Name
A1	$B = 0$ if $B \neq 1$	A1′	$B = 1$ if $B \neq 0$	Binary field
A2	$\overline{0} = 1$	A2′	$\overline{1} = 0$	NOT
A3	$0 \bullet 0 = 0$	A3′	$1 + 1 = 1$	AND/OR
A4	$1 \bullet 1 = 1$	A4′	$0 + 0 = 0$	AND/OR
A5	$0 \bullet 1 = 1 \bullet 0 = 0$	A5′	$1 + 0 = 0 + 1 = 1$	AND/OR

Dual: Exchange: • and +
0 and 1

COMBINATIONAL LOGIC DESIGN

Basic Boolean Theorems

	Theorem
T1	$B \bullet 1 = B$
T2	$B \bullet 0 = 0$
T3	$B \bullet B = B$
T4	$\overline{\overline{B}} = B$
T5	$B \bullet \overline{B} = 0$

Basic Boolean Theorems: Duals

	Theorem		Dual	Name
T1	$B \bullet 1 = B$	T1′	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	T2′	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	T3′	$B + B = B$	Idempotency
T4		$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	T5′	$B + \overline{B} = 1$	Complements

Dual: Exchange: • and +
0 and 1