Chapter 1

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CPE100: Digital Logic Design I

Section 1004: Dr. Morris

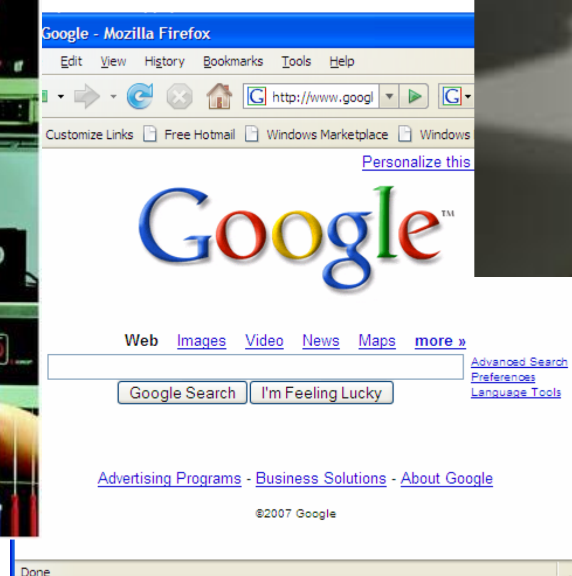
From Zero to One

Background: Digital Logic Design

- How have digital devices changed the world?
- How have digital devices changed *your* life?

Background

- Digital Devices have revolutionized our world
  - Internet, cell phones, rapid advances in medicine, etc.
- The semiconductor industry has grown from $21 billion in 1985 to over $300 billion in 2015



The Game Plan

- Purpose of course:
  - Learn the principles of digital design
  - Learn to systematically debug increasingly complex designs

Chapter 1: Topics

- The Art of Managing Complexity
- The Digital Abstraction
- Number Systems
- Addition
- Binary Codes
- Signed Numbers
- Logic Gates
- Logic Levels
- CMOS Transistors
- Power Consumption

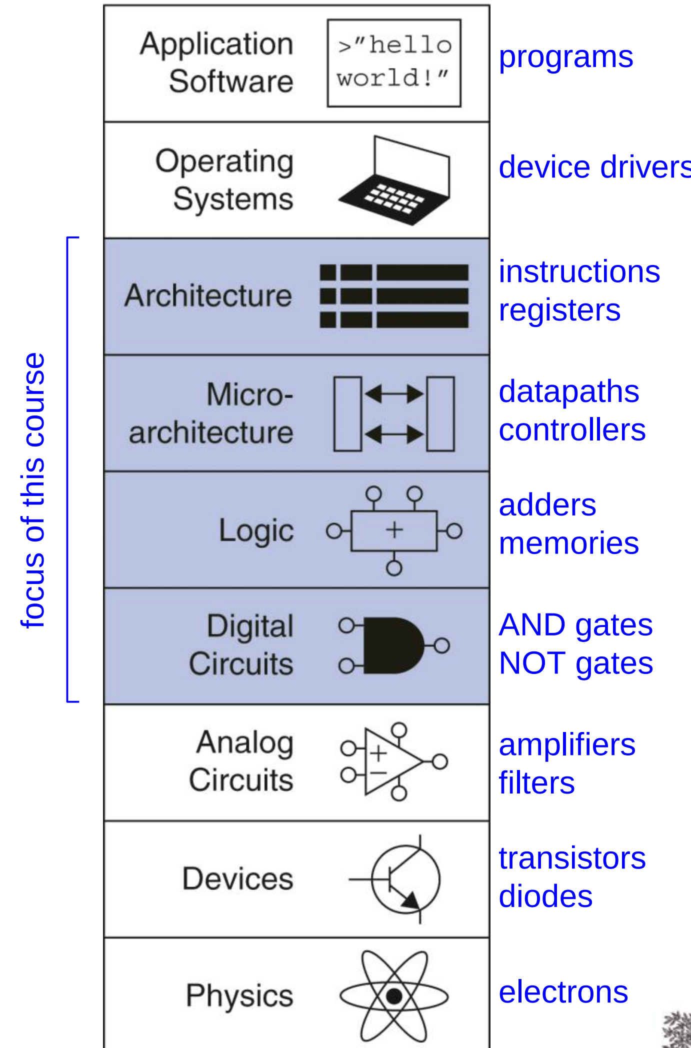
The Art of Managing Complexity

- Abstraction
- Discipline
- The Three –y's
  - Hierarch**y**
  - Modularit**y**
  - Regularit**y**

Abstraction

- What is abstraction?
  - Hiding details when they are not important

- Electronic computer abstraction
  - Different levels with different building blocks



Discipline

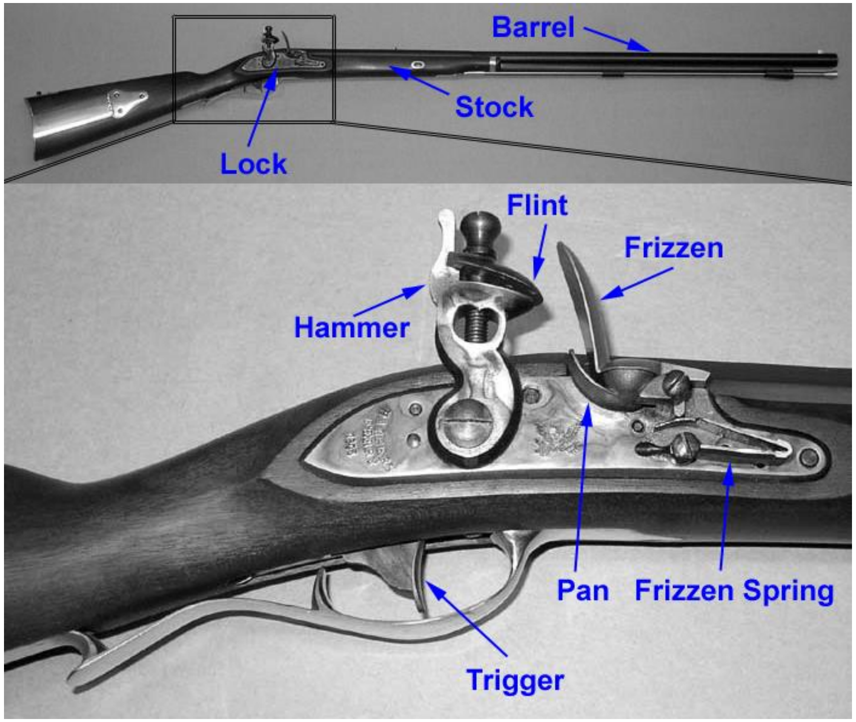
- Intentionally restrict design choices
- Example: Digital discipline
  - Discrete voltages (0 V, 5 V) instead of continuous (0V – 5V)
  - Simpler to design than analog circuits – can build more sophisticated systems
  - Digital systems replacing analog predecessors:
    - i.e., digital cameras, digital television, cell phones, CDs

The Three –y's

- Hierarchy
  - A system divided into modules and submodules
- Modularity
  - Having well-defined functions and interfaces
- Regularity
  - Encouraging uniformity, so modules can be easily reused

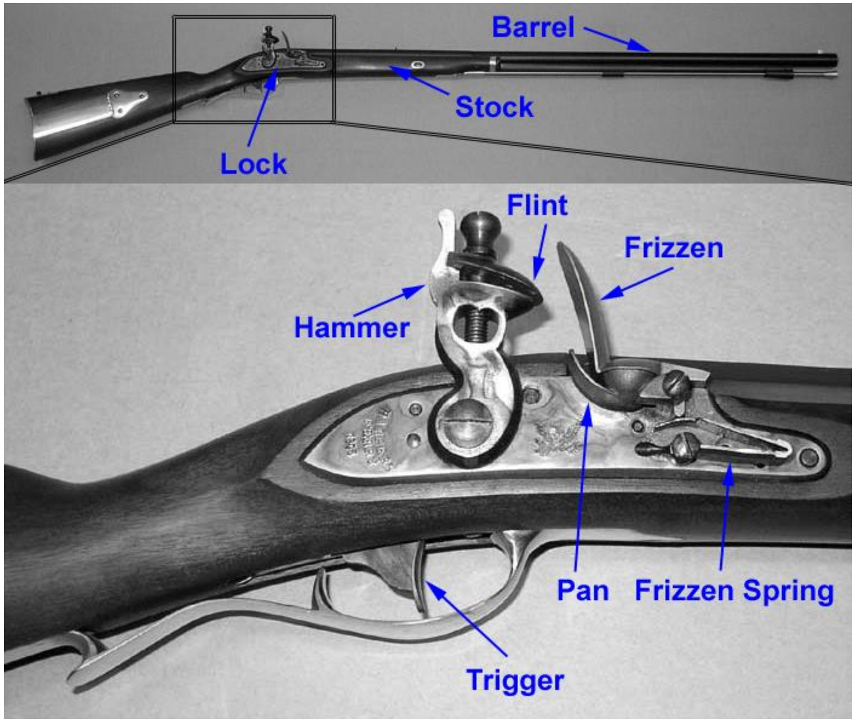
Example: Flintlock Rifle

- Hierarchy
  - Three main modules: Lock, stock, and barrel
  - Submodules of lock: Hammer, flint, frizzen, etc.



Example Flintlock Rifle

- Modularity
  - Function of stock: mount barrel and lock
  - Interface of stock: length and location of mounting pins
- Regularity
  - Interchangeable parts



The Art of Managing Complexity

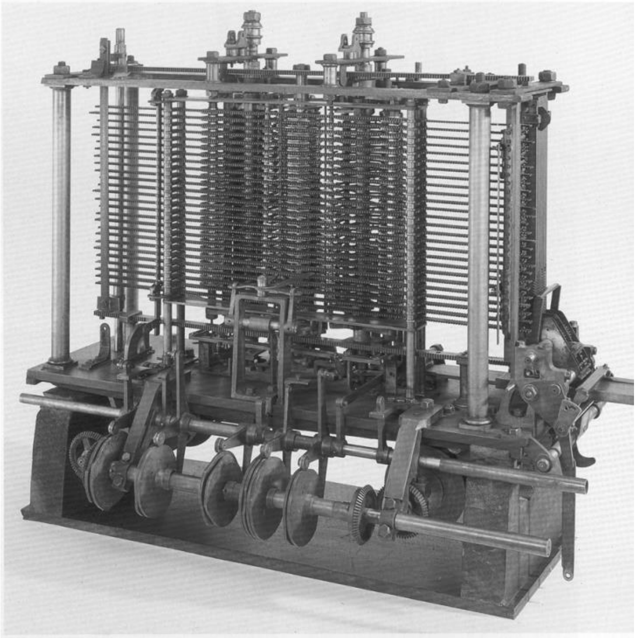
- Abstraction
- Discipline
- The Three –y's
  - Hierarch**y**
  - Modularit**y**
  - Regularit**y**

The Digital Abstraction

- Most physical variables are **continuous**
  - Voltage on a wire (1.33 V, 9 V, 12.2 V)
  - Frequency of an oscillation (60 Hz, 33.3 Hz, 44.1 kHz)
  - Position of mass (0.25 m, 3.2 m)
- Digital abstraction considers **discrete subset** of values
  - 0 V, 5 V
  - "0", "1"

The Analytical Engine

- Designed by Charles Babbage from 1834 – 1871
- Considered to be the first digital computer
- Built from mechanical gears, where each gear represented a discrete value (0-9)
- Babbage died before it was finished



Digital Discipline: Binary Values

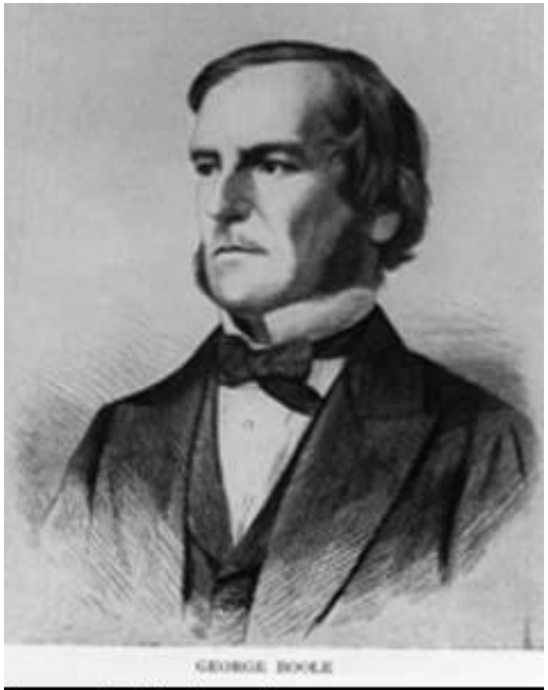
- Two discrete values
  - 1 and 0
    - 1 = TRUE = HIGH = ON
    - 0 = FALSE = LOW = OFF
- How to represent 1 and 0
  - Voltage levels, rotating gears, fluid levels, etc.
- Digital circuits use voltage levels to represent 1 and 0
  - Bit = binary digit
    - Represents the status of a digital signal (2 values)

Why Digital Systems?

- Easier to design
- Fast
- Can overcome noise
- Error detection/correction

George Boole, 1815-1864

- Born to working class parents
- Taught himself mathematics and joined the faculty of Queen’s College in Ireland
- Wrote An Investigation of the Laws of Thought (1854)
- Introduced binary variables
- Introduced the three fundamental logic operations: AND, OR, and NOT



Scanned at the American Institute of Physics

Number Systems

- Decimal
  - Base 10
- Binary
  - Base 2
- Hexadecimal
  - Base 16

Decimal Numbers

- Base 10 (our everyday number system)

1000's Column
100's Column
10's Column
1's Column

$$5374_{10} = 5 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$$

Five Thousand | Three Hundred | Seven Tens | Four Ones

Base 10

Binary Numbers

- Base 2 (computer number system)

8's Column
4's Column
2's Column
1's Column

$$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

One Eight | One Four | Zero Two | One One

Base 2

FROM ZERO TO ONE

Powers of Two

- $2^0 =$
- $2^1 =$
- $2^2 =$
- $2^3 =$
- $2^4 =$
- $2^5 =$
- $2^6 =$
- $2^7 =$
- $2^8 =$
- $2^9 =$
- $2^{10} =$
- $2^{11} =$
- $2^{12} =$
- $2^{13} =$
- $2^{14} =$
- $2^{15} =$

Powers of Two

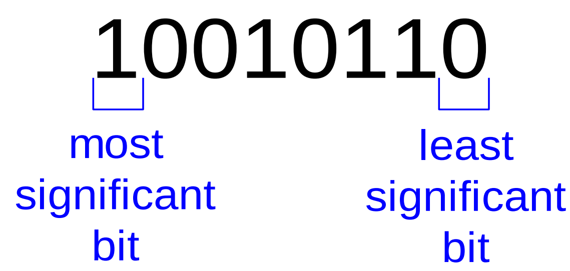
- $2^0 = 1$
- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- $2^7 = 128$
- $2^8 = 256$
- $2^9 = 512$
- $2^{10} = 1024$
- $2^{11} = 2048$
- $2^{12} = 4096$
- $2^{13} = 8192$
- $2^{14} = 16384$
- $2^{15} = 32768$
- Handy to memorize up to 2^{10}

FROM ZERO TO ONE

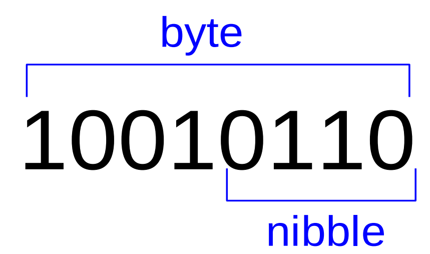
ELSEVIER

Bits, Bytes, Nibbles …

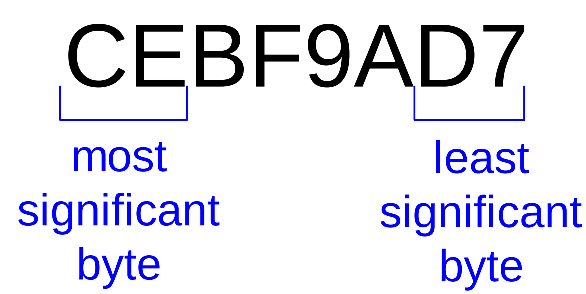
- Bits



- Bytes = 8 bits
- Nibble = 4 bits



- Words = 32 bits
  - Hex digit to represent nibble



Decimal to Binary Conversion

- Two Methods:
- Method 1: Find largest power of 2 that fits, subtract and repeat
- Method 2: Repeatedly divide by 2, remainder goes in next most significant bit

D2B: Method 1

- Find largest power of 2 that fits, subtract, repeat

53_{10}

D2B: Method 1

- Find largest power of 2 that fits, subtract, repeat

53_{10}	32×1
$53-32 = 21$	16×1
$21-16 = 5$	4×1
$5-4 = 1$	1×1

$$= 110101_2$$

FROM ZERO TO ONE

D2B: Method 2

- Repeatedly divide by 2, remainder goes in next most significant bit

$$53_{10} =$$

D2B: Method 2

- Repeatedly divide by 2, remainder goes in next most significant bit

$53_{10} =$	$53/2 = 26$ R1	LSB
	$26/2 = 13$ R0	
	$13/2 = 6$ R1	
	$6/2 = 3$ R0	
	$3/2 = 1$ R1	
	$1/2 = 0$ R1	MSB

$$= 110101_2$$

Number Conversion

- Binary to decimal conversion
  - Convert 10011_2 to decimal

$$16 \times 1 + 8 \times 0 + 4 \times 0 + 2 \times 1 + 1 \times 1 = 19_{10}$$

- Decimal to binary conversion
  - Convert 47_{10} to binary

$$32 \times 1 + 16 \times 0 + 8 \times 1 + 4 \times 1 + 2 \times 1 + 1 \times 1 = 101111_2$$

D2B Example

- Convert 75_{10} to binary

D2B Example

- Convert 75_{10} to binary

$$75_{10} = 64 + 8 + 2 + 1 = 1001011_2$$

- Or

$$\begin{aligned}
75/2 &= 37 \quad R1 \\
37/2 &= 18 \quad R1 \\
18/2 &= 9 \quad R0 \\
9/2 &= 4 \quad R1 \\
4/2 &= 2 \quad R0 \\
2/2 &= 1 \quad R0 \\
1/2 &= 0 \quad R1
\end{aligned}$$

Binary Values and Range

- N-digit decimal number
  - How many values?
  - Range?
- Example:
  3-digit decimal number
  - Possible values
  - Range

Binary Values and Range

- N-digit decimal number
  - How many values?
    - 10^N
  - Range?
    - $[0, 10^N - 1]$

- Example:
  3-digit decimal number
  - Possible values
    - $10^3 = 1000$
  - Range
    - $[0, 999]$

Binary Values and Range

- N-bit binary number
  - How many values?
  - Range?
- Example:
  3-bit binary number
  - Possible values
  - Range

Binary Values and Range

- N-bit binary number
  - How many values?
    - 2^N
  - Range?
    - $[0, 2^N - 1]$

- Example:
  3-bit binary number
  - Possible values
    - $2^3 = 8$
  - Range
    - $[0, 7] = [000_2, 111_2]$

Binary Values and Range

- N-digit decimal number
  - How many values?
    - 10^N
  - Range?
    - $[0, 10^N - 1]$
- Example:
  3-digit decimal number
  - Possible values
    - $10^3 = 1000$
  - Range
    - $[0, 999]$

- N-bit binary number
  - How many values?
    - 2^N
  - Range?
    - $[0, 2^N - 1]$
- Example:
  3-bit binary number
  - Possible values
    - $2^3 = 8$
  - Range
    - $[0, 7] = [000_2, 111_2]$

Hexadecimal Numbers

- Base 16 number system
- Shorthand for binary
  - Four binary digits (4-bit binary number) is a single hex digit

Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	
8	8	
9	9	
A	10	
B	11	
C	12	
D	13	
E	14	
F	15	

Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
  - Convert $4AF_{16}$ (also written 0x4AF) to binary

- Hexadecimal to decimal conversion:
  - Convert 0x4AF to decimal

FROM ZERO TO ONE

ELSEVIER

Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
  - Convert $4AF_{16}$ (also written 0x4AF) to binary
    - 0x4AF $= 0100\ 1010\ 1111_2$
- Hexadecimal to decimal conversion:
  - Convert 0x4AF to decimal
    - $4 \times 16^2 + 10 \times 16^1 + 15 \times 16^0 = 1199_{10}$

Number Systems

- Popular

• Decimal	Base 10
• Binary	Base 2
• Hexadecimal	Base 16

- Others

• Octal	Base 8
• Any other base	

Octal Numbers

- Same as hex with one less binary digit

Octal Digit	Decimal Equivalent	Binary Equivalent
0	0	000
1	1	001
2	2	010
3	3	011
4	4	100
5	5	101
6	6	110
7	7	111

Number Systems

- In general, an N-digit number $\{a_{N-1}a_{N-2} \ldots a_1a_0\}$ of base R in decimal equals
  - $a_{N-1}R^{N-1} + a_{N-2}R^{N-2} + \cdots + a_1R^1 + a_0R^0$
- Example: 4-digit {5173} of base 8 (octal)

Number Systems

- In general, an N-digit number $\{a_{N-1}a_{N-2} \ldots a_1 a_0\}$ of base R in decimal equals
  - $a_{N-1}R^{N-1} + a_{N-2}R^{N-2} + \cdots + a_1 R^1 + a_0 R^0$
- Example: 4-digit $\{5173\}$ of base 8 (octal)
  - $5 \times 8^3 + 1 \times 8^2 + 7 \times 8^1 + 3 \times 8^0 = 2683_{10}$

FROM ZERO TO ONE

ELSEVIER

Decimal to Octal Conversion

- Remember two methods for D2B conversion
  - 1: remove largest multiple; 2: repeated divide
- Convert 29_{10} to octal

Decimal to Octal Conversion

- Remember two methods for D2B conversion
  - 1: remove largest multiple; 2: repeated divide
- Convert 29_{10} to octal
- Method 2

29/8	=3	R5	lsb
3/8	=0	R3	msb

$$29_{10} = 35_8$$

Decimal to Octal Conversion

- Remember two methods for D2B conversion
  - 1: remove largest multiple; 2: repeated divide
- Convert 29_{10} to octal
- Method 1

29 $\quad$ 8×3=24

29-24=5

$$29_{10} = 24 + 5 = 3 \times 8^1 + 5 \times 8^0 = 35_8$$

  - Or (better scalability)

$$29_{10} = 16 + 8 + 4 + 1 = 11101_2 = 35_8$$

Octal to Decimal Conversion

- Convert 163_8 to decimal

Octal to Decimal Conversion

- Convert 163_8 to decimal

  - $163_8 = 1 \times 8^2 + 6 \times 8^1 + 3$
  - $163_8 = 64 + 48 + 3$
  - $163_8 = 115_{10}$

Recap: Binary and Hex Numbers

- Example 1: Convert 83_{10} to hex

- Example 2: Convert 01101011_2 to hex and decimal

- Example 3: Convert 0xCA3 to binary and decimal

Recap: Binary and Hex Numbers

- Example 1: Convert 83_{10} to hex
  - $83_{10} = 64 + 16 + 2 + 1 = 1010011_2$
  - $1010011_2 = 101\ 0011_2 = 53_{16}$
- Example 2: Convert 01101011_2 to hex and decimal
  - $01101011_2 = 0110\ 1011_2 = 6B_{16}$
  - 0x6B $= 6 \times 16^1 + 11 \times 16^0 = 96 + 11 = 107$
- Example 3: Convert 0xCA3 to binary and decimal
  - 0xCA3 $= 1100\ 1010\ 0011_2$
  - 0xCA3 $= 12 \times 16^2 + 10 \times 16^1 + 3 \times 16^0 = 3235_{10}$

Large Powers of Two

- 2^{10} = 1 kilo ≈ 1000 (1024)
- 2^{20} = 1 mega ≈ 1 million (1,048,576)
- 2^{30} = 1 giga ≈ 1 billion (1,073,741,824)
- 2^{40} = 1 tera ≈ 1 trillion (1,099,511,627,776)

Large Powers of Two: Abbreviations

- 2^{10} = 1 kilo ≈ 1000 (1024)

  for example: 1 kB = 1024 Bytes

  1 kb = 1024 bits

- 2^{20} = 1 mega ≈ 1 million (1,048,576)

  for example: 1 MiB, 1 Mib (1 megabit)

- 2^{30} = 1 giga ≈ 1 billion (1,073,741,824)

  for example: 1 GiB, 1 Gib

Estimating Powers of Two

- What is the value of 2^{24}?

- How many values can a 32-bit variable represent?

Estimating Powers of Two

- What is the value of 2^{24}?
  - **$2^4 \times 2^{20} \approx$ 16 million**

- How many values can a 32-bit variable represent?
  - **$2^2 \times 2^{30} \approx$ 4 billion**

Binary Codes

Another way of representing decimal numbers

Example binary codes:

- Weighted codes
  - Binary Coded Decimal (BCD) (8-4-2-1 code)
  - 6-3-1-1 code
  - 8-4-2-1 code (simple binary)
- Gray codes
- Excess-3 code
- 2-out-of-5 code

Binary Codes

Decimal #	8-4-2-1 (BCD)	6-3-1-1	Excess-3	2-out-of-5	Gray
0	0000	0000	0011	00011	0000
1	0001	0001	0100	00101	0001
2	0010	0011	0101	00110	0011
3	0011	0100	0110	01001	0010
4	0100	0101	0111	01010	0110
5	0101	0111	1000	01100	1110
6	0110	1000	1001	10001	1010
7	0111	1001	1010	10010	1011
8	1000	1011	1011	10100	1001
9	1001	1100	1100	11000	1000

Each code combination represents **a single decimal digit.**

Weighted Codes

- Weighted codes: each bit position has a given weight
  - Binary Coded Decimal (BCD) (8-4-2-1 code)
    - **Example:** $\mathbf{726}_{10}$ = $\mathbf{0111\ 0010\ 0110}_{BCD}$
  - 6-3-1-1 code
    - **Example:** 1001 (6-3-1-1 code) = 1×6 + 0×3 + 0×1 + 1×1
    - **Example:** $\mathbf{726}_{10}$ = $\mathbf{1001\ 0011\ 1000}_{6311}$
- BCD numbers are used to represent fractional numbers exactly (vs. floating point numbers – which can't - see Chapter 5)

FROM ZERO TO ONE

Weighted Codes

Decimal #	8-4-2-1 (BCD)	6-3-1-1
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0100
4	0100	0101
5	0101	0111
6	0110	1000
7	0111	1001
8	1000	1011
9	1001	1100

- **BCD Example:**

726_{10} = **0111 0010 0110**$_{BCD}$

- **6-3-1-1 code Example:**

726_{10} = **1001 0011 1000**$_{6311}$

Excess-3 Code

Decimal #	Excess-3
0	0011
1	0100
2	0101
3	0110
4	0111
5	1000
6	1001
7	1010
8	1011
9	1100

- Add 3 to number, then represent in binary
  - **Example:** 5_{10} = 5+3 = 8 = $\mathbf{1000_2}$
- Also called a **biased** number
- Excess-3 codes (also called XS-3) were used in the 1970's to ease arithmetic

- **Excess-3 Example:**

$726_{10} = 1010\ 0101\ 1001_{xs3}$

2-out-of-5 Code

Decimal #	2-out-of-5
0	00011
1	00101
2	00110
3	01001
4	01010
5	01100
6	10001
7	10010
8	10100
9	11000

- 2 out of the 5 bits are 1
- Used for error detection:
  - If more or less than 2 of 5 bits are 1, error

Gray Codes

Decimal #	Gray
0	0000
1	0001
2	0011
3	0010
4	0110
5	1110
6	1010
7	1011
8	1001
9	1000

- Next number differs in only one bit position
  - **Example:** 000, 001, 011, 010, 110, 111, 101, 100
- **Example use:** Analog-to-Digital (A/D) converters. Changing 2 bits at a time (i.e., 011 →100) could cause large inaccuracies.

Addition

- Decimal

$$\begin{array}{r} 3734 \\ +\ \ 5168 \\ \hline \end{array}$$

- Binary

$$\begin{array}{r} 1011 \\ +\ \ 0011 \\ \hline \end{array}$$

Addition

- Decimal

```
   11  ← carries
   3734
+  5168
-------
   8902
```

- Binary

```
   1011
+  0011
-------
```

Addition

- Decimal

```
    11  ← carries
    3734
  + 5168
  ------
    8902
```

- Binary

```
    11  ← carries
    1011
  + 0011
  ------
    1110
```

Binary Addition Examples

- Add the following 4-bit binary numbers

$$\begin{array}{r} 1001 \\ +\ 0101 \\ \hline \end{array}$$

- Add the following 4-bit binary numbers

$$\begin{array}{r} 1011 \\ +\ 0110 \\ \hline \end{array}$$

Binary Addition Examples

- Add the following 4-bit binary numbers

```
     1
   1001
+  0101
-------
   1110
```

- Add the following 4-bit binary numbers

```
   1011
+  0110
-------
```

Binary Addition Examples

- Add the following 4-bit binary numbers

```
      1
   1001
+  0101
-------
   1110
```

- Add the following 4-bit binary numbers

```
   111
   1011
+  0110
-------
  10001
```

Overflow!

Overflow

- Digital systems operate on a **fixed number of bits**
- Overflow: when result is too big to fit in the available number of bits
- See previous example of 11 + 6

Signed Binary Numbers

- Sign/Magnitude Numbers
- Two's Complement Numbers

Sign/Magnitude

- 1 sign bit, *N*-1 magnitude bits
- Sign bit is the most significant (left-most) bit
  - **Positive number:** sign bit = 0
  - **Negative number:** sign bit = 1

$$A : \{a_{N-1}, a_{N-2}, \cdots a_2, a_1, a_0\}$$

$$A = (-1)^{a_{n-1}} \sum_{i=0}^{n-2} a_i 2^i$$

- Example, 4-bit sign/magnitude representations of ± 6:
  - +6 =
  - -6 =
- Range of an *N*-bit sign/magnitude number:
  -

FROM ZERO TO ONE

ELSEVIER

Sign/Magnitude

- 1 sign bit, *N*-1 magnitude bits
- Sign bit is the most significant (left-most) bit
  - **Positive number:** sign bit = 0
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$$A:\{a_{N-1},a_{N-2},\cdots a_2,a_1,a_0\}$$

$$A=(-1)^{a_{n-1}}\sum_{i=0}^{n-2}a_i 2^i$$

- Example, 4-bit sign/magnitude representations of ± 6:
  - +6 = **0110**
  - -6 = **1110**
- Range of an *N*-bit sign/magnitude number:
  - **$[-(2^{N-1}-1), 2^{N-1}-1]$**

Sign/Magnitude Numbers

- Problems:
  - Addition doesn't work, for example -6 + 6:

$$\begin{array}{r} 1110 \\ +\ 0110 \\ \hline \end{array}$$

  - Two representations of 0 (± 0):
    - (+0) =
    - (−0) =

Sign/Magnitude Numbers

- Problems:
  - Addition doesn't work, for example -6 + 6:

$$\begin{array}{r} 1110 \\ +\ 0110 \\ \hline 10100 \end{array}$$ (wrong!)

  - Two representations of 0 (± 0):
    - $(+0) = 0000$
    - $(-0) = 1000$

Two's Complement Numbers

- Don't have same problems as sign/magnitude numbers:
  - Addition works
  - Single representation for 0
- Range of representable numbers not symmetric
  - One extra negative number

Two's Complement Numbers

- msb has value of -2^{N-1}

$$A = a_{n-1}\left(-2^{n-1}\right) + \sum_{i=0}^{n-2} a_i 2^i$$

- The most significant bit still indicates the sign (1 = negative, 0 = positive)
- Range of an *N*-bit two's comp number?

- Most positive 4-bit number?
- Most negative 4-bit number?

Two's Complement Numbers

- msb has value of -2^{N-1}

$$A = a_{n-1}\left(-2^{n-1}\right) + \sum_{i=0}^{n-2} a_i 2^i$$

- The most significant bit still indicates the sign (1 = negative, 0 = positive)
- Range of an *N*-bit two's comp number?
  - $[-(2^{N-1}), 2^{N-1} - 1]$
- Most positive 4-bit number? 0111
- Most negative 4-bit number? 1000

FROM ZERO TO ONE

ELSEVIER

"Taking the Two's Complement"

- **Flips the sign** of a two's complement number
- Method:
  1. Invert the bits
  2. Add 1
- Example: Flip the sign of $3_{10} = 0011_2$

"Taking the Two's Complement"

- **Flips the sign** of a two's complement number
- Method:
  1. Invert the bits
  2. Add 1
- Example: Flip the sign of $3_{10} = 0011_2$
  1. **1100**
  2. **+ 1**

  $1101 = -3_{10}$

Two's Complement Examples

- Take the two's complement of $6_{10} = 0110_2$

- What is the decimal value of the two's complement number 1001_2?

Two's Complement Examples

- Take the two's complement of $6_{10} = 0110_2$
  1. 1001
  2. + 1

  $1010_2 = -6_{10}$

- What is the decimal value of the two's complement number 1001_2?
  1. 0110
  2. + 1

  $0111_2 = 7_{10}$, so $1001_2 = -7_{10}$

Two's Complement Addition

- Add 6 + (-6) using two's complement numbers

```
    0110
+   1010
--------
```

- Add -2 + 3 using two's complement numbers

```
    1110
+   0011
--------
```

Two's Complement Addition

- Add 6 + (-6) using two's complement numbers

```
 111
   0110
+  1010
-------
  10000
```

- Add -2 + 3 using two's complement numbers

```
   1110
+  0011
-------
```

Two's Complement Addition

- Add 6 + (-6) using two's complement numbers

```
   111
    0110
+   1010
--------
   10000
```

- Add -2 + 3 using two's complement numbers

```
   111
    1110
+   0011
--------
   10001
```

Increasing Bit Width

- Extend number from N to M bits (M > N) :
  - Sign-extension
  - Zero-extension

Sign-Extension

- Sign bit copied to msb's
- Number value is same

- Example 1
  - 4-bit representation of 3 = 0011
  - 8-bit sign-extended value:
- Example 2
  - 4-bit representation of -7 = 1001
  - 8-bit sign-extended value:

Sign-Extension

- Sign bit copied to msb's
- Number value is same

- Example 1
  - 4-bit representation of 3 = 0011
  - 8-bit sign-extended value: 00000011
- Example 2
  - 4-bit representation of -7 = 1001
  - 8-bit sign-extended value: 11111001

Zero-Extension

- Zeros copied to msb's
- Value changes for negative numbers

- Example 1
  - 4-bit value = 0011_2
  - 8-bit zero-extended value:
- Example 2
  - 4-bit value = 1001
  - 8-bit zero-extended value:

Zero-Extension

- Zeros copied to msb's
- Value changes for negative numbers

- Example 1
  - 4-bit value = 0011_2
  - 8-bit zero-extended value: 00000011
- Example 2
  - 4-bit value = 1001
  - 8-bit zero-extended value: 00001001

Zero-Extension

- Zeros copied to msb's
- Value changes for negative numbers

- Example 1
  - 4-bit value = $0011_2 = 3_{10}$
  - 8-bit zero-extended value: $00000011 = 3_{10}$
- Example 2
  - 4-bit value = $1001 = -7_{10}$
  - 8-bit zero-extended value: $00001001 = 9_{10}$

Number System Comparison

Number System	Range
Unsigned	$[0, 2^N-1]$
Sign/Magnitude	$[-(2^{N-1}-1), 2^{N-1}-1]$
Two's Complement	$[-2^{N-1}, 2^{N-1}-1]$

For example, 4-bit representation:

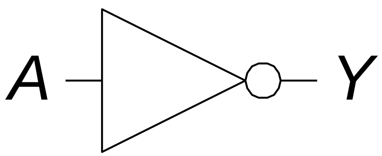
	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Unsigned									0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Two's Complement	1000	1001	1010	1011	1100	1101	1110	1111	0000	0001	0010	0011	0100	0101	0110	0111								
Sign/Magnitude		1111	1110	1101	1100	1011	1010	1001	0000 1000	0001	0010	0011	0100	0101	0110	0111								

Logic Gates

- **Perform logic functions:**
  - inversion (NOT), AND, OR, NAND, NOR, etc.
- **Single-input:**
  - NOT gate, buffer
- **Two-input:**
  - AND, OR, XOR, NAND, NOR, XNOR
- **Multiple-input**

Single-Input Logic Gates

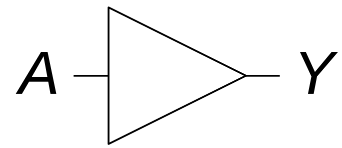
NOT



$$Y = \overline{A}$$

A	Y
0	
1	

BUF



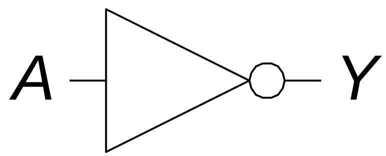
$$Y = A$$

A	Y
0	
1	

Single-Input Logic Gates

- Bubble on wire indicates inversion

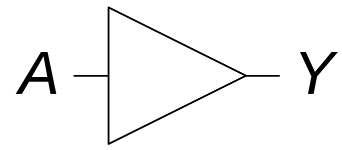
NOT



$Y = \overline{A}$

A	Y
0	1
1	0

BUF



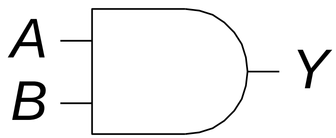
$Y = A$

A	Y
0	0
1	1

- Note: bar over variable indicates complement (invert value)

Two-Input Logic Gates

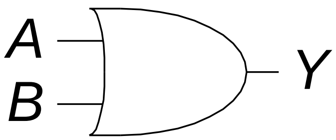
AND



$$Y = AB$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

OR

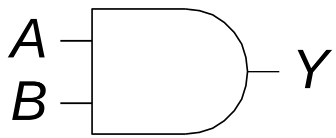


$$Y = A + B$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

Two-Input Logic Gates

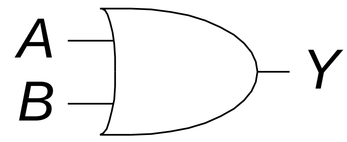
AND



$Y = AB$

A	*B*	*Y*
0	0	0
0	1	0
1	0	0
1	1	1

OR

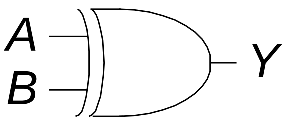


$Y = A + B$

A	*B*	*Y*
0	0	0
0	1	1
1	0	1
1	1	1

More Two-Input Logic Gates

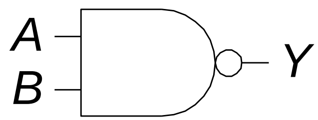
XOR



$$Y = A \oplus B$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

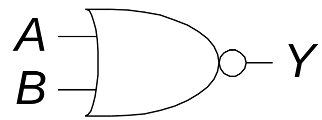
NAND



$$Y = \overline{AB}$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

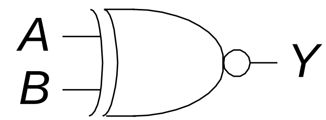
NOR



$$Y = \overline{A + B}$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

XNOR

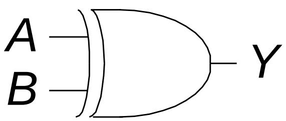


$$Y = \overline{A \oplus B}$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

More Two-Input Logic Gates

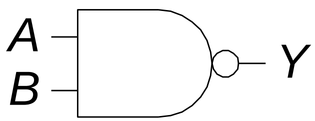
XOR



$$Y = A \oplus B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

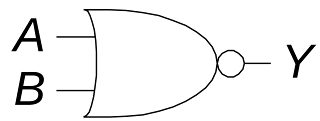
NAND



$$Y = \overline{AB}$$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

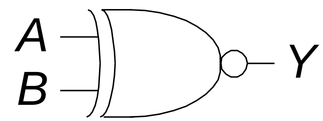
NOR



$$Y = \overline{A + B}$$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

XNOR

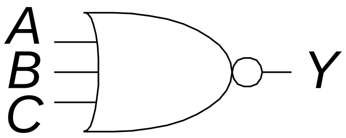


$$Y = \overline{A \oplus B}$$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Multiple-Input Logic Gates

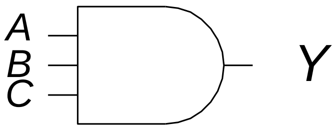
NOR3



$Y = \overline{A+B+C}$

A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

AND3

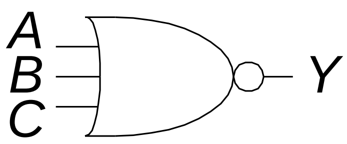


$Y = ABC$

A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Multiple-Input Logic Gates

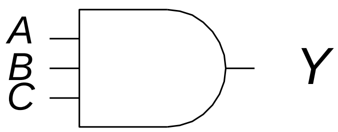
NOR3



$Y = \overline{A+B+C}$

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

AND3



$Y = ABC$

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

- Multi-input XOR = Odd parity (one for odd input=1)

Logic Levels

- Discrete voltages represent 1 and 0
- For example:
  - 0 = *ground* (GND) or 0 volts
  - 1 = V_{DD} or 5 volts
- What about 4.99 volts? Is that a 0 or a 1?
- What about 3.2 volts?

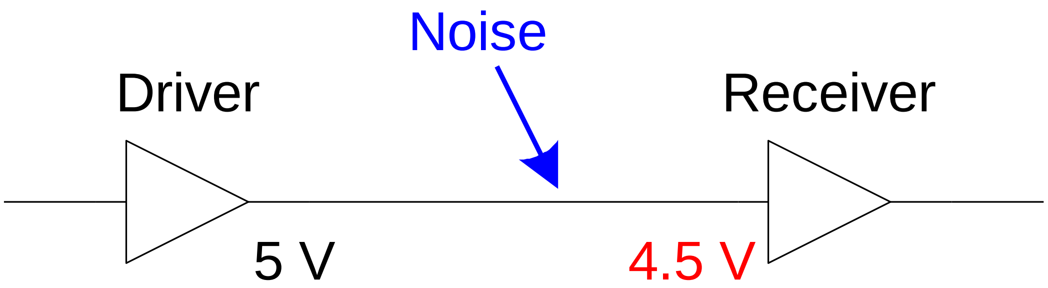
Logic Levels

- Must have *range* of voltages for 1 and 0
- Different ranges for inputs and outputs to allow for *noise*

What is Noise?

- Anything that degrades the signal
  - E.g., resistance, power supply noise, coupling to neighboring wires, etc.

- Example: a gate (driver) outputs 5 V but, because of resistance in a long wire, receiver gets 4.5 V



The Static Discipline

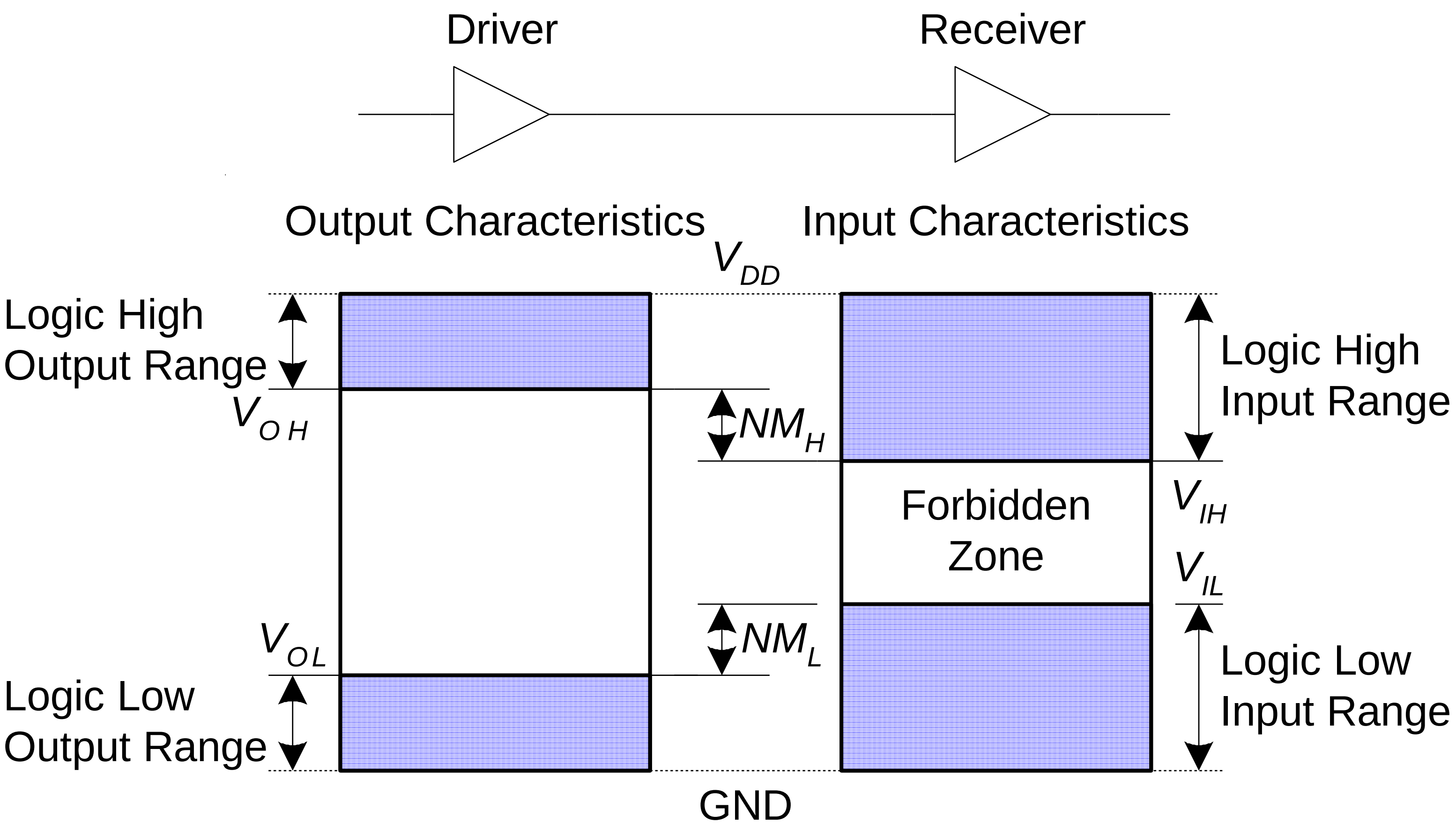
- With logically valid inputs, every circuit element must produce logically valid outputs

- Use limited ranges of voltages to represent discrete values

Real Logic Levels

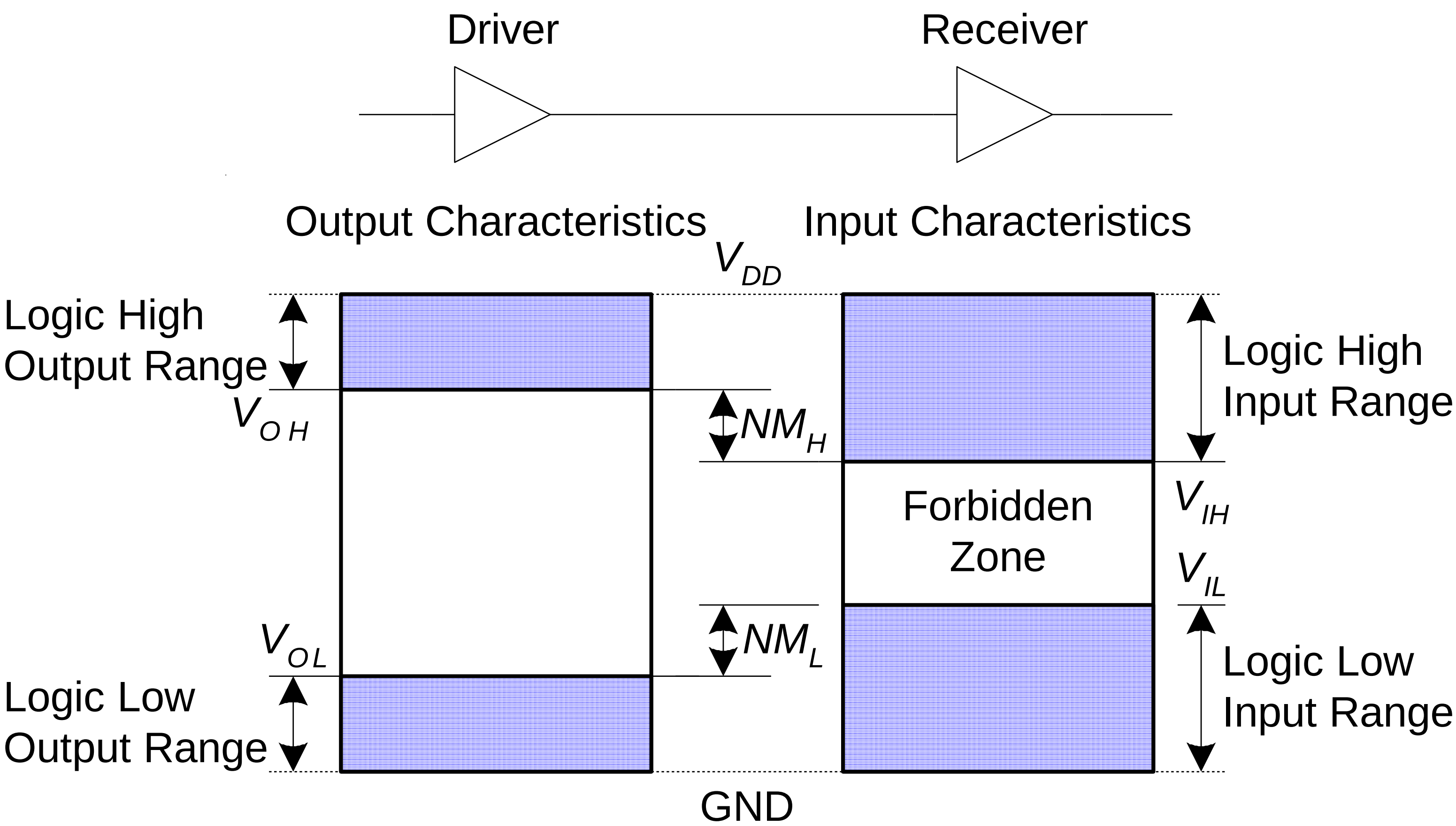
- Want driver to output "clean" high/low and receiver to handle noisy high/low

Real Logic Levels



- Want driver to output "clean" high/low and receiver to handle noisy high/low

Real Logic Levels



$$NM_H = V_{OH} - V_{IH}$$

$$NM_L = V_{IL} - V_{OL}$$

V_{DD} Scaling

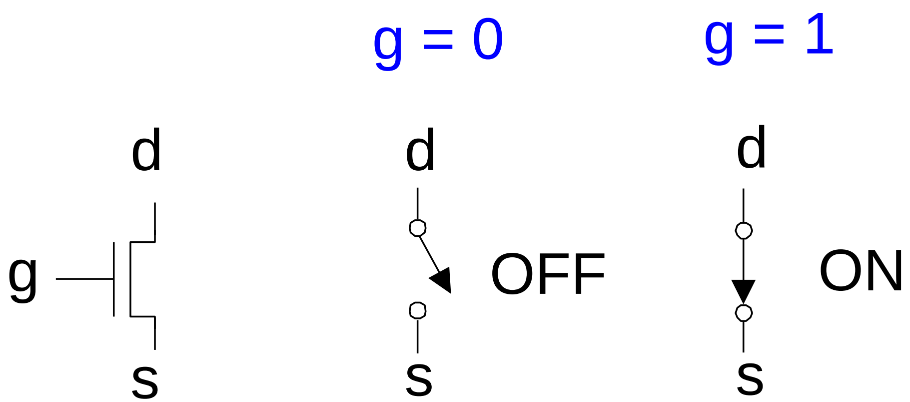
- In 1970's and 1980's, V_{DD} = 5 V
- V_{DD} has dropped
  - 3.3 V, 2.5 V, 1.8 V, 1.5 V, 1.2 V, 1.0 V, …
  - Avoid frying tiny transistors
  - Save power
- Be careful connecting chips with different supply voltages
  - Easy to fry if not careful

Logic Family Examples

Logic Family	V_{DD}	V_{IL}	V_{IH}	V_{OL}	V_{OH}
TTL	5 (4.75 - 5.25)	0.8	2.0	0.4	2.4
CMOS	5 (4.5 - 6)	1.35	3.15	0.33	3.84
LVTTL	3.3 (3 - 3.6)	0.8	2.0	0.4	2.4
LVCMOS	3.3 (3 - 3.6)	0.9	1.8	0.36	2.7

Transistors

- Logic gates built from transistors
- Simple model: 3-ported voltage-controlled switch
  - 2 ports connected depending on voltage of 3rd
  - d and s are connected (ON) when g is 1

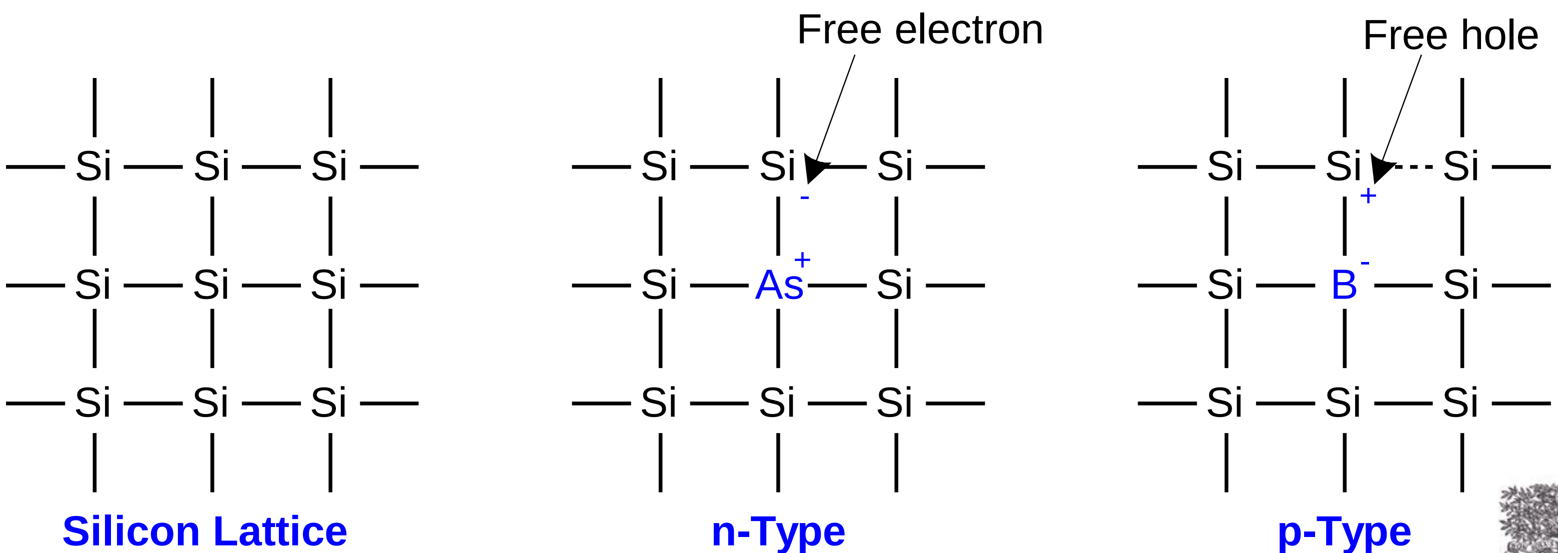


Robert Noyce, 1927-1990

- Nicknamed "Mayor of Silicon Valley"
- Cofounded Fairchild Semiconductor in 1957
- Cofounded Intel in 1968
- Co-invented the integrated circuit

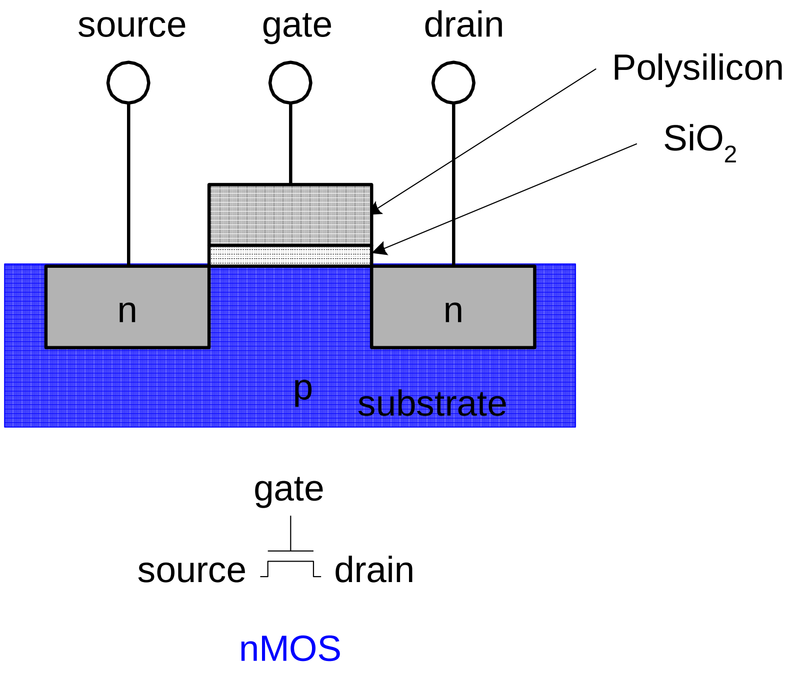
Silicon

- Transistors built from silicon, a semiconductor
- Pure silicon is a poor conductor (no free charges)
- Doped silicon is a good conductor (free charges)
  - n-type (free negative charges, electrons)
  - p-type (free positive charges, holes)



MOS Transistors

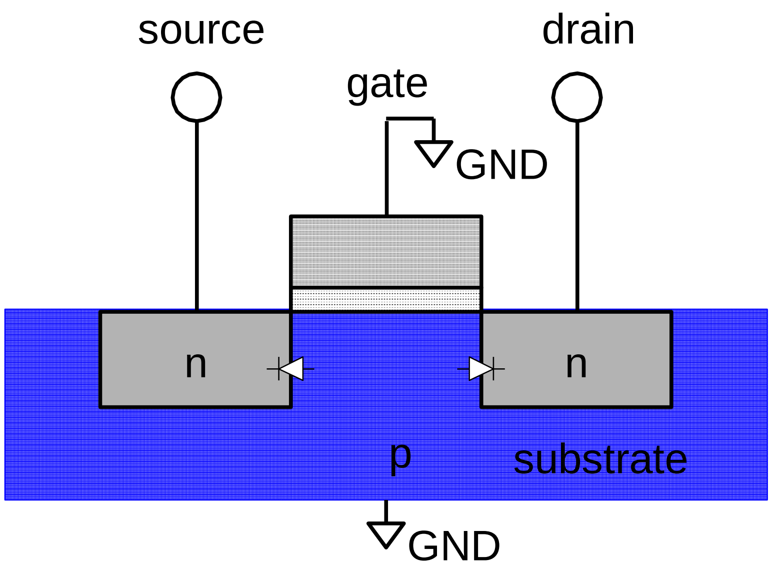
- Metal oxide silicon (MOS) transistors:
  - Polysilicon (used to be metal) gate
  - Oxide (silicon dioxide) insulator
  - Doped silicon

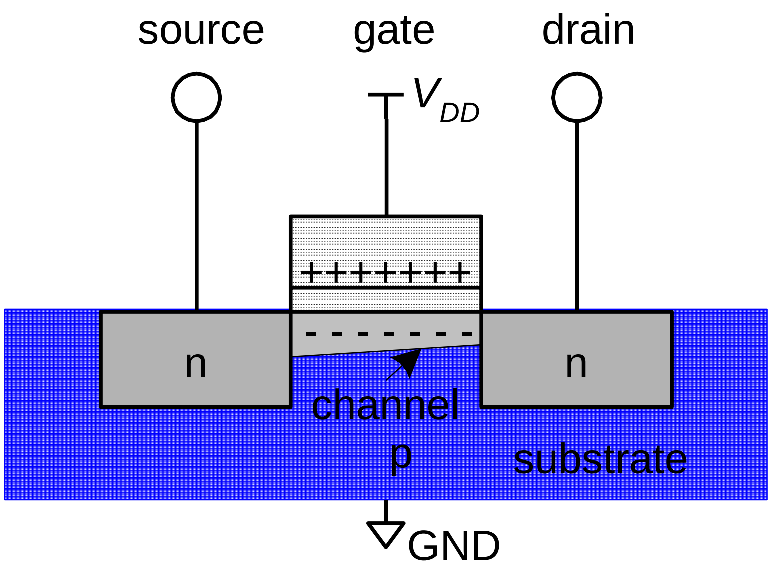


nMOS Transistors

- Gate = 0
- OFF (no connection between source and drain)

- Gate = 1
- ON (channel between source and drain)

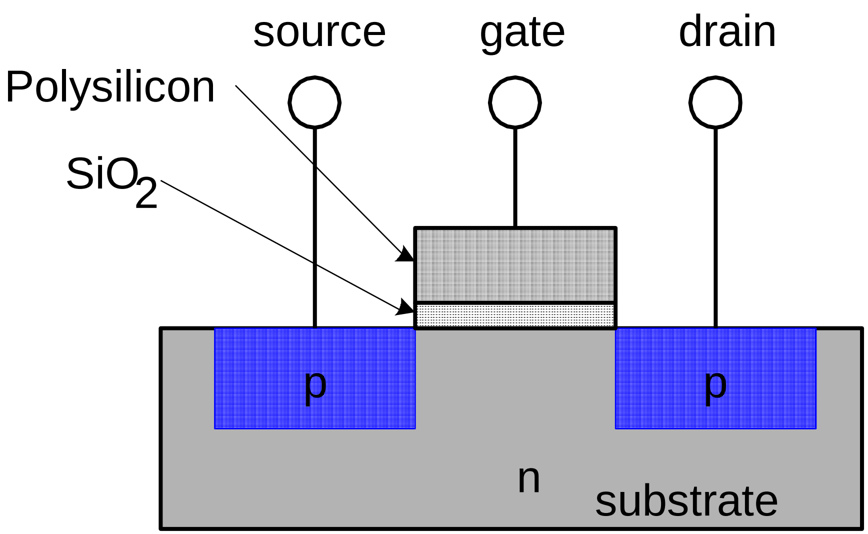




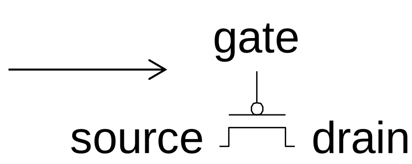
Diode connection from p to n doped area
→ current cannot travel from n→p

pMOS Transistors

- pMOS transistor is opposite of nMOS
  - ON when Gate = 0
  - OFF when Gate = 1

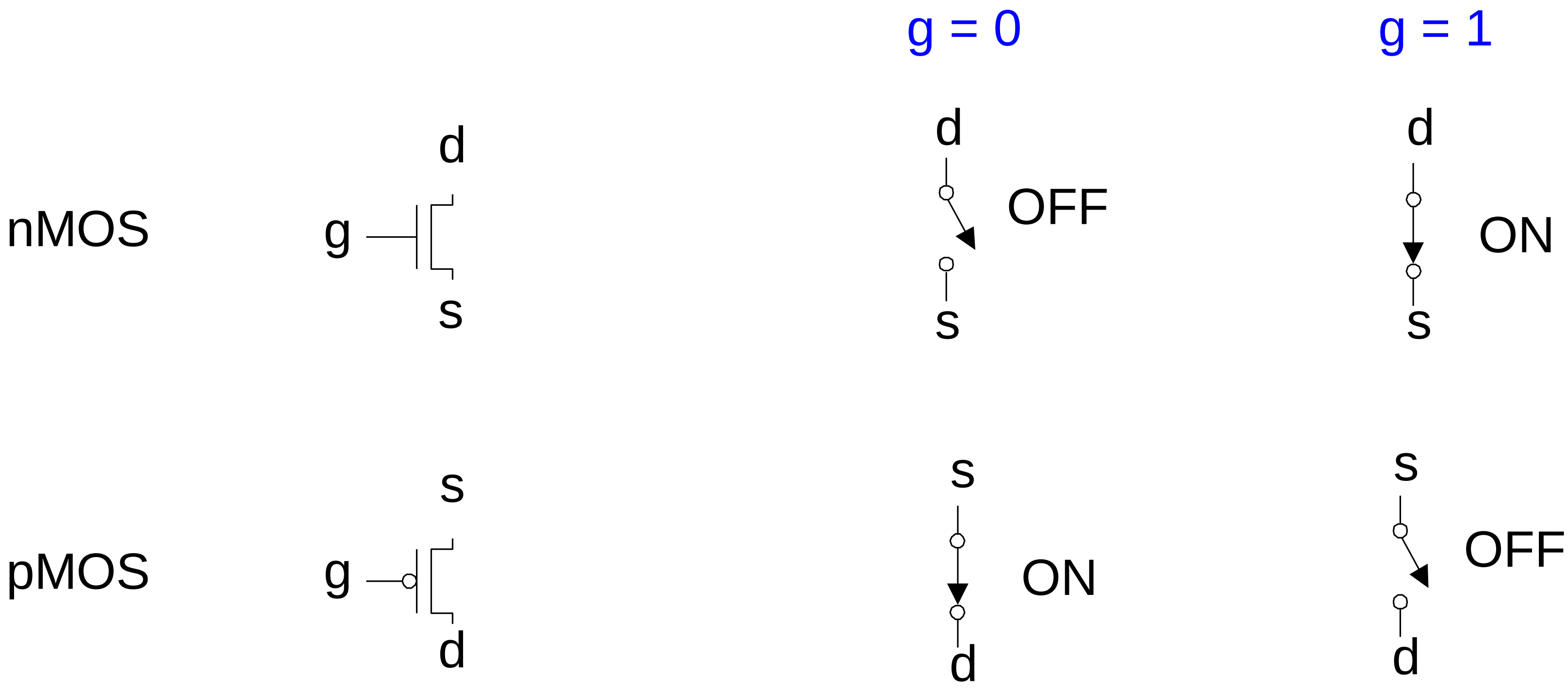


Note bubble on gate
to indicate on when low



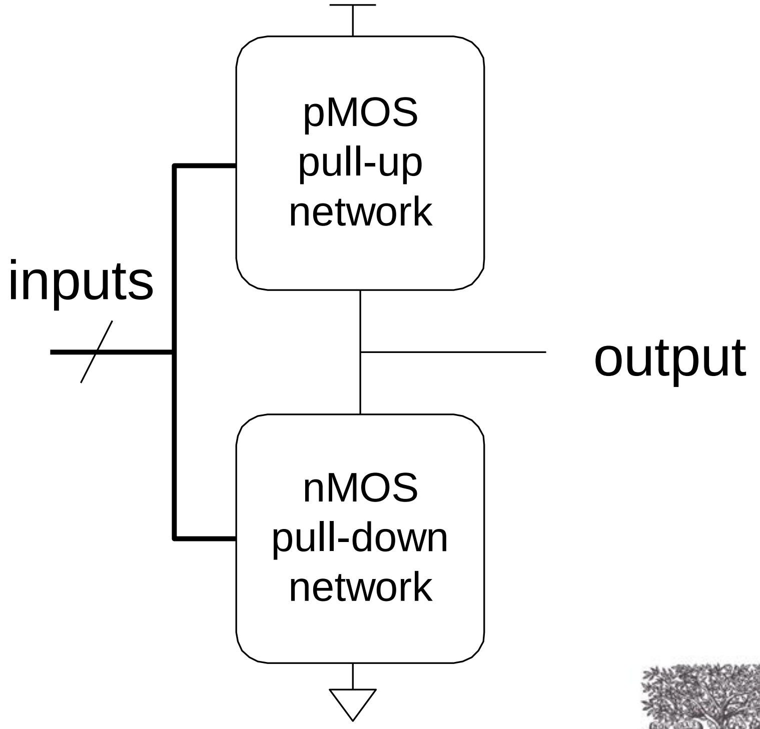
Transistor Function

- Voltage controlled switch



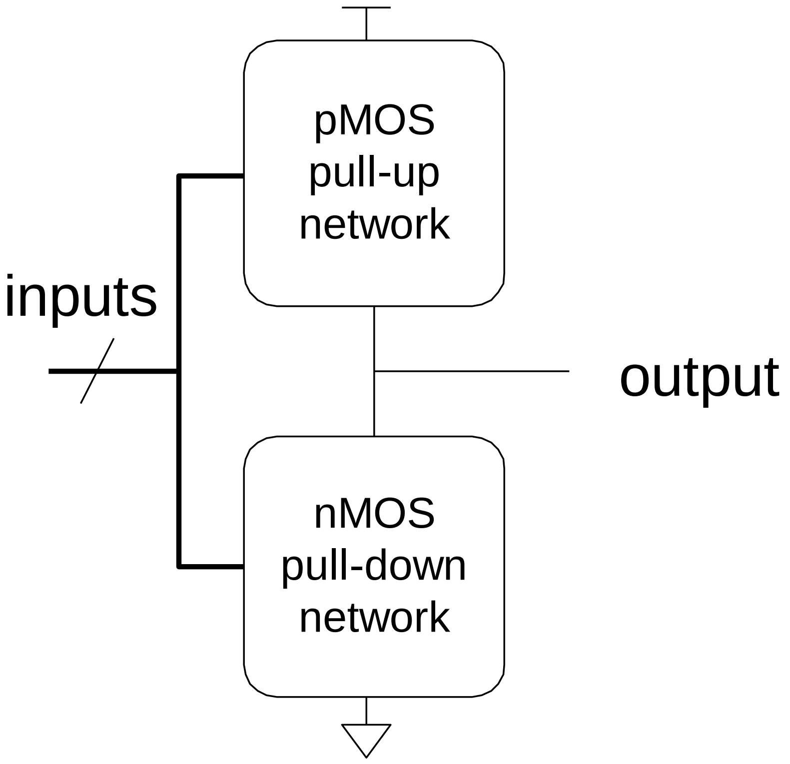
Transistor Composition

- nMOS: pass good 0's
  - Connect source to GND
  - "Pull down" transistor
- pMOS: pass good 1's
  - Connect source to VDD
  - "Pull up" transistor
- Build logic gates from composition
  - CMOS = complementary MOS



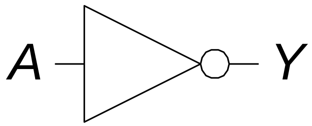
CMOS Gate Structure

- Pull-up pMOS network connects to V_{DD}
- Pull-down nMOS network connects to GND
- Use series and parallel connections to implement gate logic



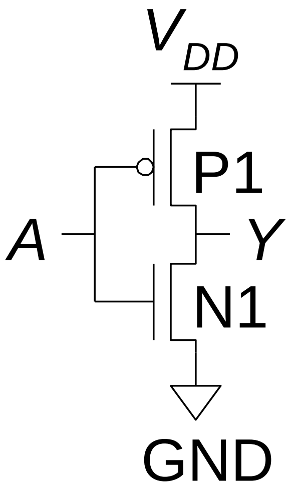
CMOS Gates: NOT Gate

NOT



$Y = \overline{A}$

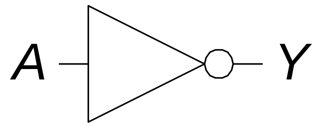
A	Y
0	1
1	0



A	P1	N1	Y
0			
1			

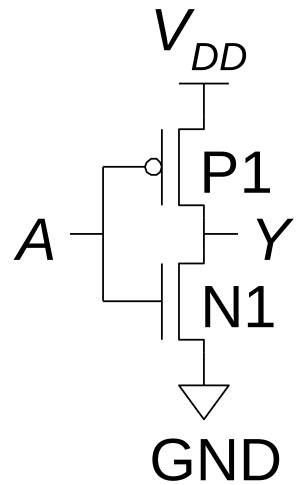
CMOS Gates: NOT Gate

NOT



$Y = \overline{A}$

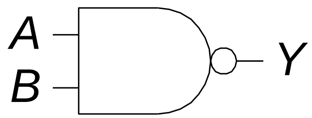
A	Y
0	1
1	0



A	P1	N1	Y
0	ON	OFF	1
1	OFF	ON	0

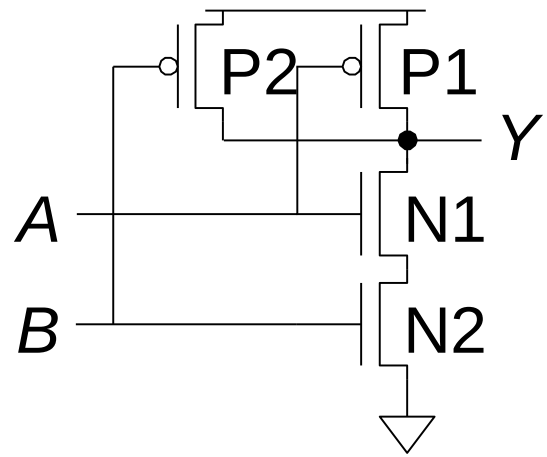
CMOS Gates: NAND Gate

NAND



$Y = \overline{AB}$

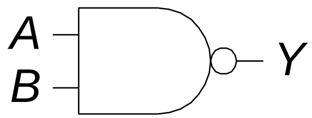
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



A	*B*	P1	P2	N1	N2	*Y*
0	0					
0	1					
1	0					
1	1					

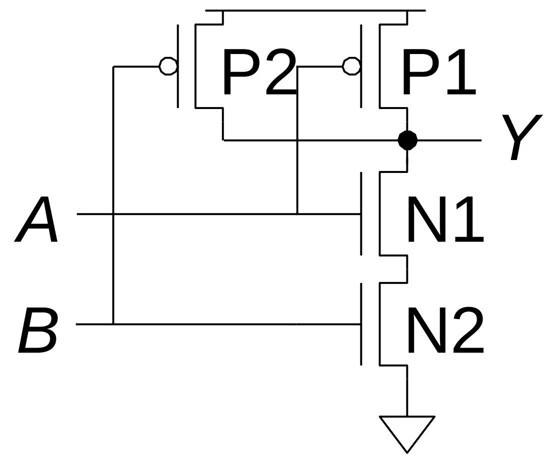
CMOS Gates: NAND Gate

NAND



$Y = \overline{AB}$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



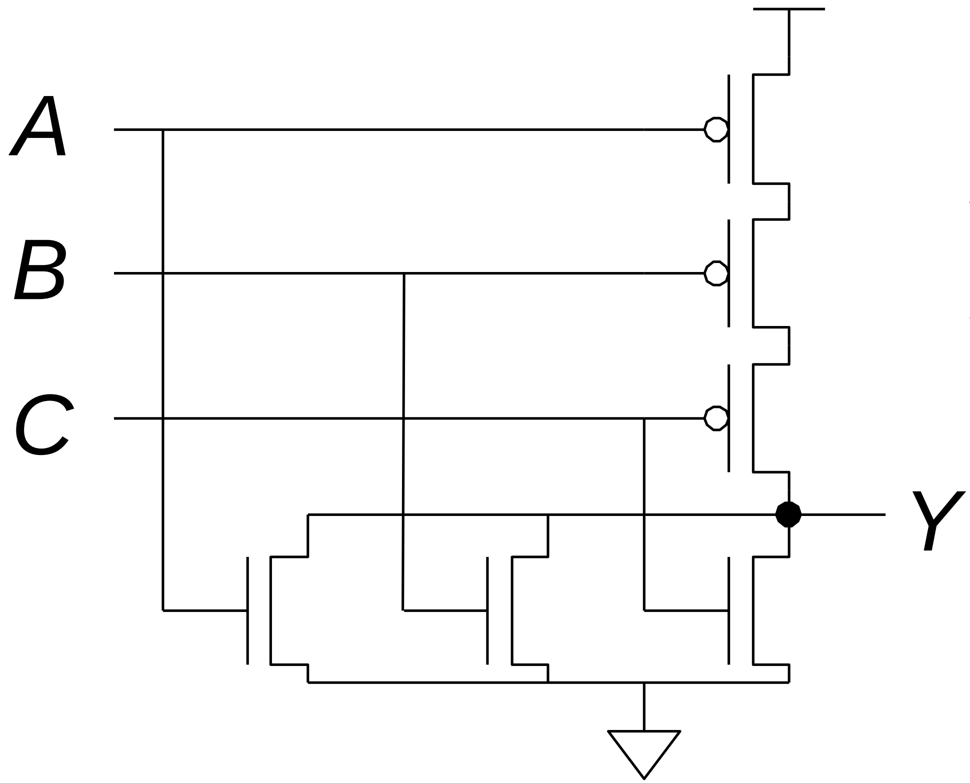
A	*B*	P1	P2	N1	N2	*Y*
0	0	ON	ON	OFF	OFF	1
0	1	ON	OFF	OFF	ON	1
1	0	OFF	ON	ON	OFF	1
1	1	OFF	OFF	ON	ON	0

CMOS Gates: NOR Gate

- How can you build three input (A, B, C) NOR gate?

CMOS Gates: NOR Gate

- How can you build three input (A, B, C) NOR gate?



Only high output when all three pMOS in series are "on" and create a path from output to V_{DD}

CMOS Gates: AND Gate

- How can you build 2 input AND gate?

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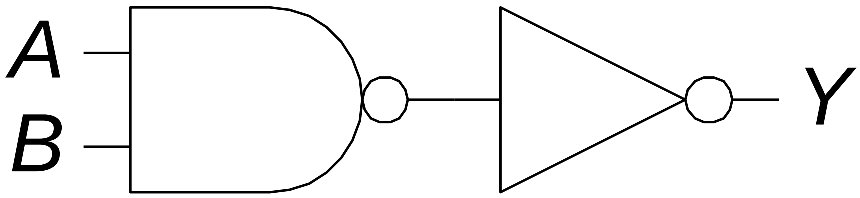
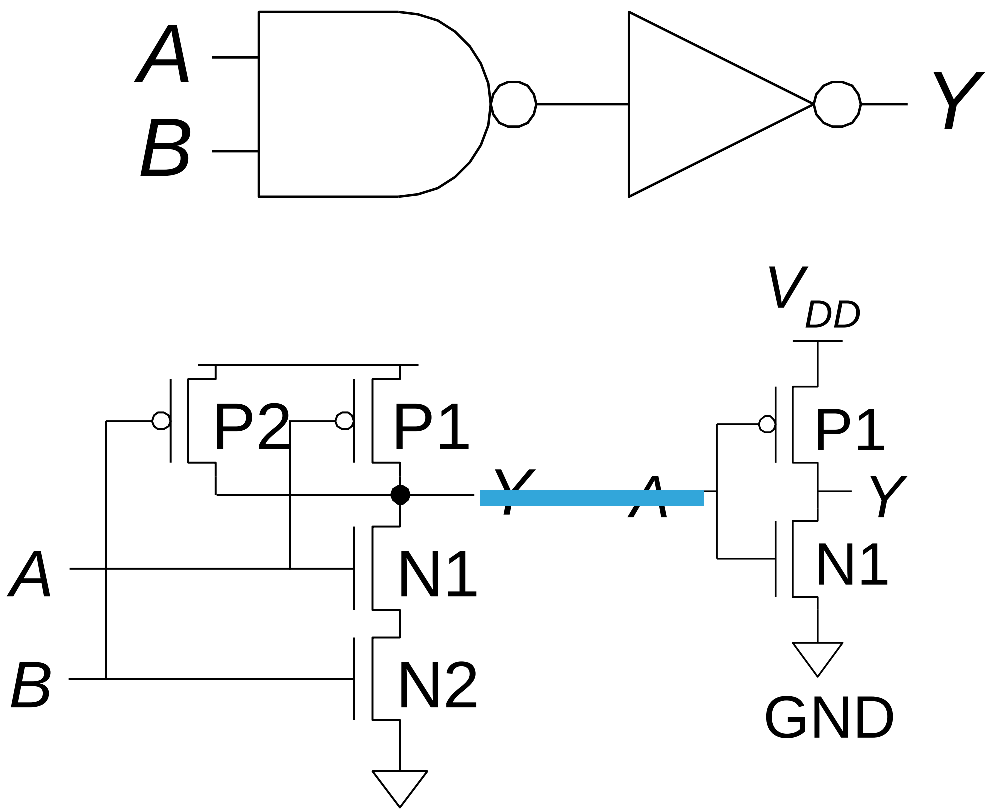


CMOS Gates: AND Gate

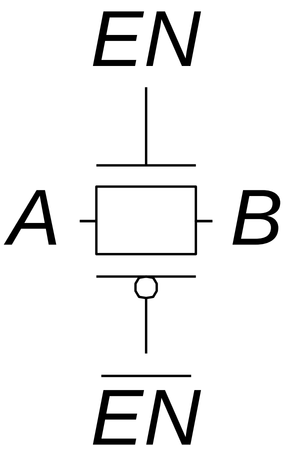
- How can you build 2 input AND gate?



Note: AND requires 2 more gates than NAND. Inverted logic is more efficient implementation.

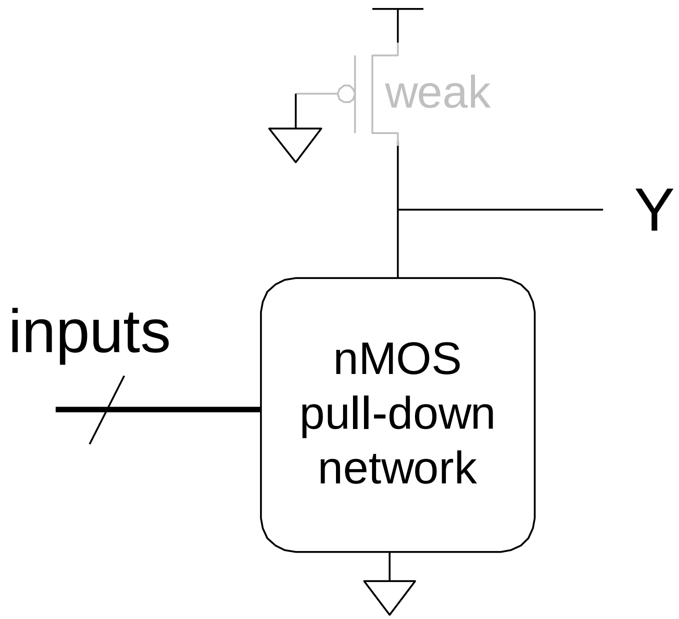
Transmission Gates

- nMOS pass 1's poorly, pMOS pass 0's poorly
- Transmission gate is for passing signal
  - Pass both 0 and 1 well
- When EN = 1, the switch is ON:
  - $\overline{EN} = 0$ and A is connected to B
- When EN = 0, the switch is OFF:
  - A is not connected to B



Psuedo-nMOS

- Replace pull-up network with weak pMOS transistor that is always on
  - pMOS gate tied to ground
- pMOS transistor: pulls output HIGH only when nMOS network not pulling it LOW



Psuedo-nMOS Example: NOR4

- How many transistors needed?

FROM ZERO TO ONE

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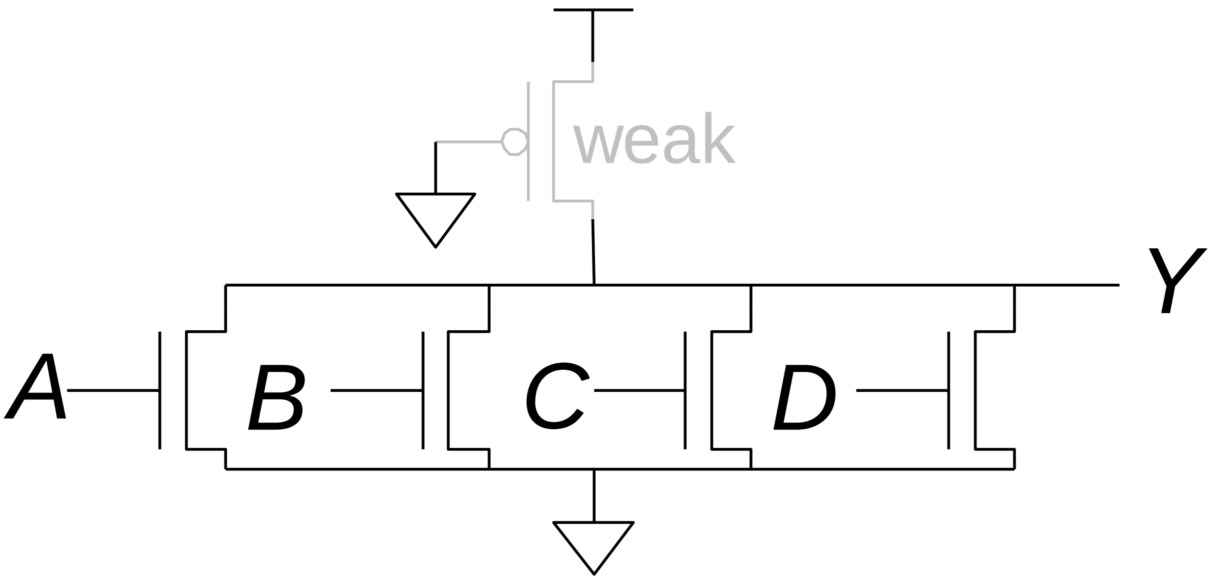
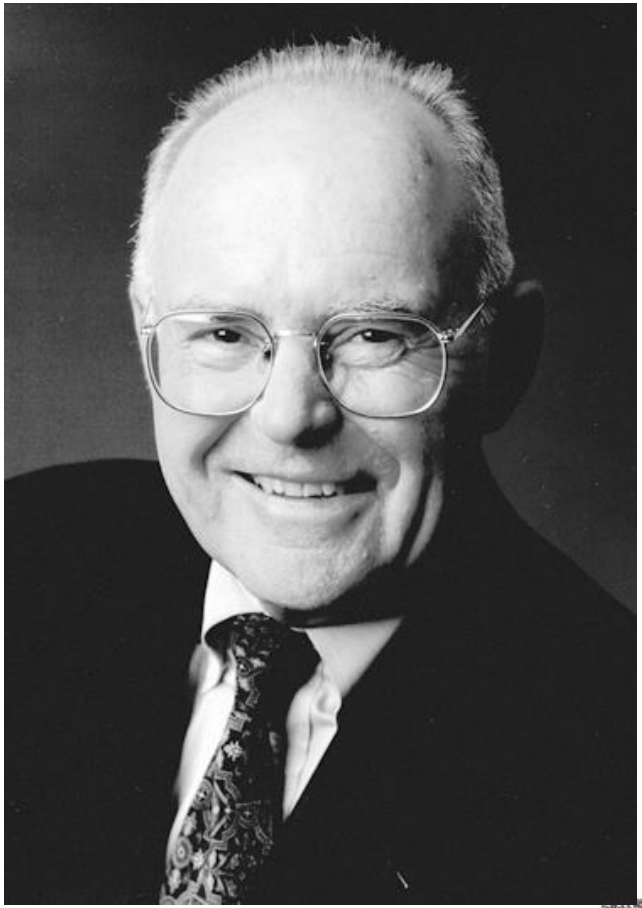
Psuedo-nMOS Example: NOR4

- How many transistors needed?
  - Only 5 since a single pMOS is used



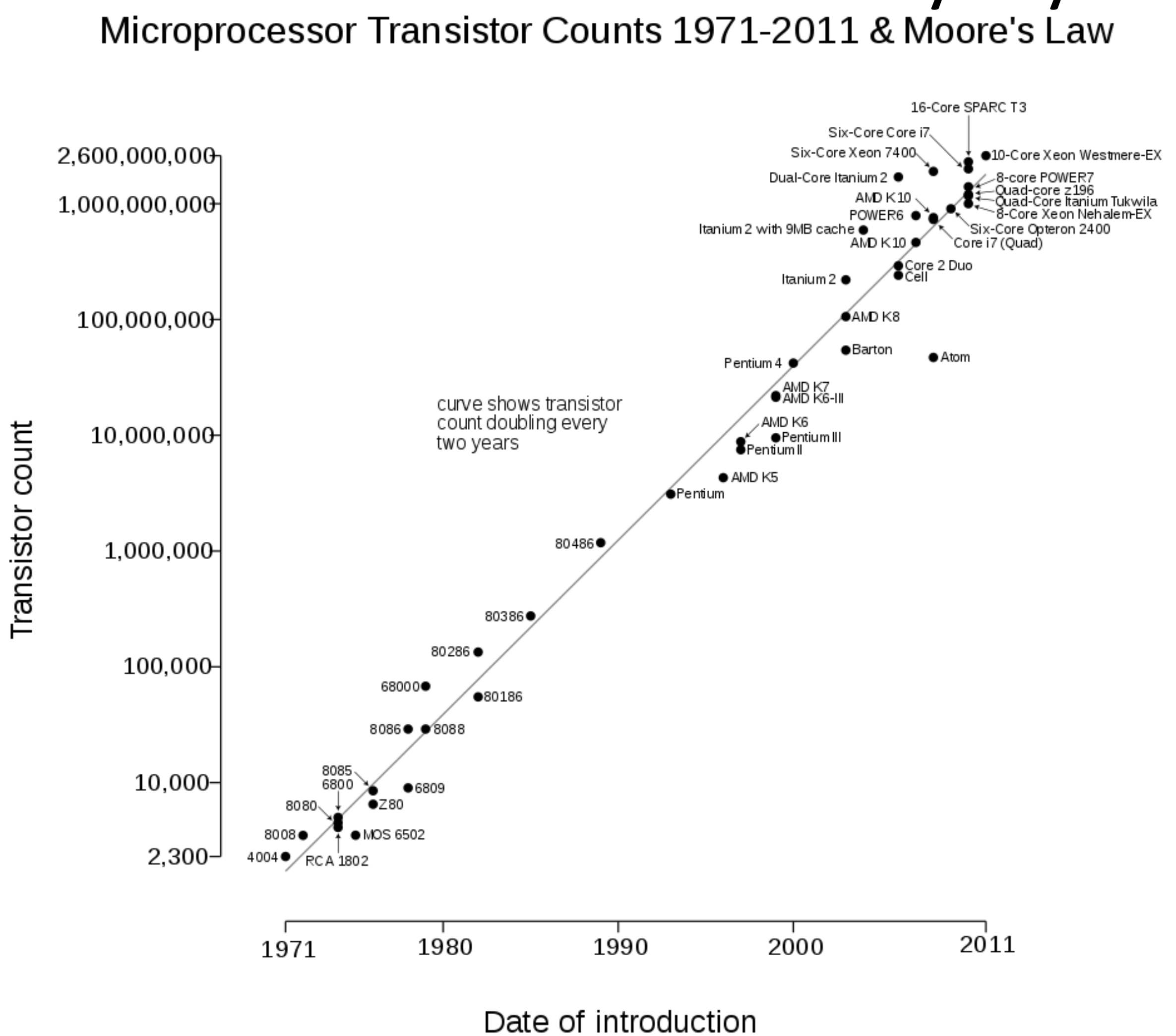
Gordon Moore, 1929-

- Cofounded Intel in 1968 with Robert Noyce.
- Moore's Law: number of transistors on a computer chip doubles every year (observed in 1965)
  - Since 1975, transistor counts have doubled every two years.

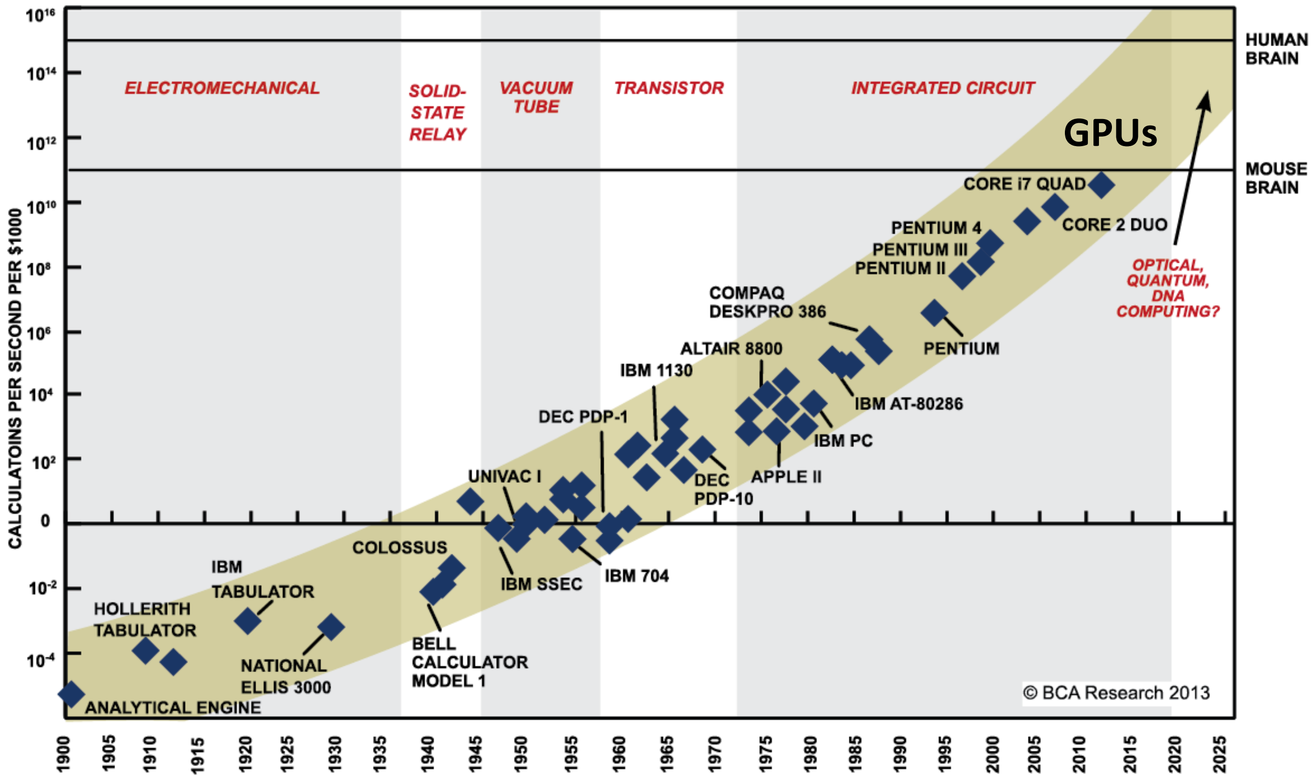


Moore's Law

- Transistor count doubles every 2 years



Moore's Law Trends



SOURCE: RAY KURZWEIL, "THE SINGULARITY IS NEAR: WHEN HUMANS TRANSCEND BIOLOGY", P.67, *THE VIKING PRESS*, 2006. DATAPOINTS BETWEEN 2000 AND 2012 REPRESENT BCA ESTIMATES.

- *"If the automobile had followed the same development cycle as the computer, a Rolls-Royce would today cost $100, get one million miles to the gallon, and explode once a year . . ."*

  – Robert Cringley

Power Consumption

- Power = Energy consumed per unit time

- Two types of power
  - Dynamic power consumption
  - Static power consumption

Dynamic Power Consumption

- Power to charge transistor gate capacitances
  - Energy required to charge a capacitance, C, to V_{DD} is CV_{DD}^2
  - Circuit running at frequency f: transistors switch (from 1 to 0 or vice versa) at that frequency
  - Capacitor is charged $f/2$ times per second (discharging from 1 to 0 is free)
- Dynamic power consumption

$$P_{dynamic} = \frac{1}{2} C V_{DD}^2 f$$

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Static Power Consumption

- Power consumed when no gates are switching
- Caused by the quiescent supply current, I_{DD} (also called the leakage current)

- Static power consumption

$$P_{static} = I_{DD}V_{DD}$$

Power Consumption Example

- Estimate the power consumption of a wireless handheld computer
  - $V\_DD =$ 1.2 V
  - $C =$ 20 nF
  - $f =$ 1 GHz
  - $I_{DD} =$ 20 mA
- Total power is sum of dynamic and static

FROM ZERO TO ONE

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Power Consumption Example

- Estimate the power consumption of a wireless handheld computer
  - $V_DD = $ 1.2 V
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- Total power is sum of dynamic and static

$$P = \frac{1}{2} C V_{DD}^2 f + I_{DD} V_{DD}$$
$$= \frac{1}{2}(20\text{ n})(1.2)^2(1\text{ G}) + (20\text{ m})(1.2)$$
$$= (14.4 + 0.024)\text{W}$$
$$= 14.4\text{ W}$$

FROM ZERO TO ONE

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