Homework #4
Due Su. 10/01

**Note:** Do not use a calculator or computer to complete the following exercises. You must show all your work and put a box around your final answer to receive credit. Messy or unreadable submissions will receive no credit.

Homework will only be accepted at the beginning of class and all pages must be stapled together.

**Total Points:** 69

1. (0 points) How long did it take you to complete the homework? This will not affect your grade (unless omitted) but it helps gauge the workload for this and future semesters. If you do not answer this question you will get -5 points.

2. (13 points) What is the function of the transistor-level circuit? Complete the truth table for the circuit.

**Solution**

(a) (3 points) \( Y = \text{NOT}(A) \)

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<th>A</th>
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(b) (5 points) \( Y = \text{NAND}(A,B,C) \)

Give output equation in POS form. \( Y = (\bar{A} + \bar{B} + \bar{C}) = \overline{ABC} \)

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(c) (5 points) \( Y = \text{OR}(A,B,C) \)

Give output in SOP form. \( Y = \overline{\overline{A}BC} + \overline{\overline{A}B\overline{C}} + \overline{AB\overline{C}} + \overline{ABC} + \overline{AB\overline{C}} + \overline{ABC} + \overline{ABC} + ABC \)

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3. (12 points) Minimize each of the following Boolean equations. Show your work and list which axiom or theorem was used in each step. The final equation should be in minimized sum-of-product (SOP) form.

**Solution**

(a) (3 points) \( Y = \overline{AB} + AB \)

\[
Y = \overline{AB} + AB \\
= B \\
\text{T10 Combining}
\]

(b) (3 points) \( Y = A\overline{BC} + ABC \)

\[
Y = A\overline{BC} + ABC \\
= AC \\
\text{T10 Combining}
\]

(c) (3 points) \( Y = AB\overline{C} + \overline{A} \)

\[
Y = AB\overline{C} + \overline{A} \\
= \overline{A} + AB\overline{C} \\
= (\overline{A} + A)(\overline{A} + B\overline{C}) \\
= (1)(\overline{A} + B\overline{C}) \\
= \overline{A} + B\overline{C} \\
\text{T1 Identity}
\]

(d) (3 points) \( Y = \overline{AB} + A \)

\[
Y = \overline{AB} + A \\
= A + \overline{AB} \\
= (A + \overline{A})(A + B) \\
= (1)(A + B) \\
= A + B \\
\text{T1 Identity}
\]
4. (10 points) Minimize each of the following Boolean equations. Show your work and list which axiom or theorem was used in each step. The final equation should be in minimized sum-of-product (SOP) form. Note: we use apostrophe “′” in this problem to indicate complement.

Solution

(a) (5 points) \( Y = ABCD' + A'B'CD + CD' \)

\[
Y = ABCD' + A'B'CD + CD' \\
= ABCD' + CD' + A'B'CD \quad \text{T6' Commutativity} \\
= CD' + A'B'CD \quad \text{T9' Covering} \\
= CD' + A'B'C \quad \text{T10 Combining} \\
= CD' + A'B'C
\]

(b) (5 points) \( Y = AB'C' + CD' + BC'D' \)

\[
Y = AB'C' + CD' + BC'D' \\
= AB'C' + CD' + BC'D' \quad \text{T9' Covering} \\
= AB'C' + CD' + BD' \quad \text{T10 Combining}
\]

5. (20 points) Use Boolean theorem’s and axioms to express the following equations in minimized SOP form. Show your work and list which axiom or theorem was used in each step.

Solution

(a) (5 points) \( Y = \overline{ABCD} = \overline{A} + \overline{B} + \overline{C} + \overline{D} \) T12 DeMorgan’s

(b) (5 points) \( Y = \overline{A + B + C + C} = \overline{A + B + C} = \overline{A}\overline{B}\overline{C} \) T12’ DeMorgan’s  
Note: this question had a typo, with two C’s. Solution for problem as stated in the homework is below.

\[
Y = \overline{A + B + C + C} \\
= \overline{A + B + C} \quad \text{T3' Idempotency} \\
= \overline{A}\overline{B}\overline{C} \quad \text{T12' DeMorgan's}
\]

(c) (5 points) \( Y = \overline{ABC + D + E} \)

\[
Y = \overline{ABC + D + E} \\
= \overline{ABC} \cdot \overline{D + E} \quad \text{T12' DeMorgan’s} \\
= ABC \cdot (\overline{D + E}) \quad \text{T4 Involution} \\
= ABCD + ABCE \quad \text{T8 Distributivity}
\]

(d) (5 points) \( Y = \overline{AB + C + DE} \)

\[
Y = \overline{AB + C + DE} \\
= \overline{AB + C} \cdot \overline{DE} \quad \text{T12' DeMorgan's} \\
= (\overline{AB + C}) \cdot (\overline{DE}) \quad \text{T4 Involution} \\
= (\overline{AB + C}) \cdot (\overline{D + E}) \quad \text{T12' DeMorgan's} \\
= (\overline{AB + C}) \cdot (\overline{D + E}) \quad \text{T4 Involution} \\
= \overline{ABD} + \overline{ABE} + CD + CE \quad \text{T8 Distributivity}
\]
6. (14 points) Consider the truth table below for logical function output $Y$ with inputs $A$, $B$, and $C$.

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Solution
(a) (3 points) Write an equation for $Y$ in canonical SOP form.

$$Y = \overline{A}B\overline{C} + A\overline{B}C + ABC + ABC$$

(b) (3 points) Write an equation for $Y$ in canonical POS form.

$$Y = (A + B + C)\overline{(A + B) + \overline{C}}\overline{(A + \overline{B} + \overline{C})}(\overline{A} + B + \overline{C})$$

(c) (3 points) Write an equation for $\overline{Y}$ in canonical SOP form.

$$\overline{Y} = \overline{A}\overline{B}\overline{C} + \overline{A}BC + \overline{ABC} + ABC$$

(d) (5 points) Write an equation for $Y = \overline{\overline{Y}}$ using the result from part (c). Use DeMorgan’s Theorem to rewrite the expression so that the final result is in canonical POS form.

$$Y = \overline{\overline{Y}} = \overline{A\overline{B}\overline{C} + ABC + ABC + ABC}$$

$$= ABC \cdot ABC \cdot ABC \cdot ABC$$

$$= (A + B + C) \cdot \overline{(A + B) + \overline{C}} \cdot (\overline{A} + \overline{B} + \overline{C}) \cdot (\overline{A} + B + \overline{C})$$

$$= (A + B + C) \cdot (A + B + \overline{C}) \cdot (A + B + \overline{C}) \cdot (A + B + \overline{C})$$