Note: Do not use a calculator or computer to complete the following exercises. You must show all your work and put a box around your final answer to receive credit. Messy or unreadable submissions will receive no credit.

You should read Appendix A.1-A.2, and A.6 to help with the last two problems.

Total Points: 89

1. (0 points) How long (in hours) did it take you to complete the homework? This will not affect your grade (unless omitted) but it helps gauge the workload for this and future semesters. If you do not answer this question you will get -5 points.

2. (4 points) Perform the one’s complement operation on the following 8-bit binary string. Provide your answer first in binary then in hexadecimal.
   
   (a) 00011001
   (b) 11001101
   (c) 00100011
   (d) 01100110

   Solutions

   Be aware, as with 2’s complement, there is the binary number representation (i.e. 1’s complement representation) and the “operation”. In particular, the ones complement operation is flipping bits, while the two’s complement is taking the negative of a binary string by flipping bits and adding one.

   (a) (1 point) 11100110 = 0xE6
   (b) (1 point) 00110010 = 0x32
   (c) (1 point) 11011100 = 0xDC
   (d) (1 point) 10110011 = 0x99

3. (9 points) Consider a 10-bit binary number. Determine how many values (1 point) can be represented and what is the range of values (2 points) for the following representations:

   (a) Unsigned binary number.
   (b) Two’s complement number.
   (c) Sign/magnitude number.

   Solution

   The number of values is $2^N$ where $N$ is the number of bits.

   (a) (3 points) Values = 1024; Range = $[0, 2^{10} - 1] = [0, 1023]$
   (b) (3 points) Values = 1024; Range = $[-2^9, 2^9 - 1] = [-512, 511]$
   (c) (3 points) Values = 1024; Range = $[-2^9 - 1, 2^9 - 1] = [-511, 511]$ (1023 is acceptable for one zero value)
4. (2 points) Suppose you are offered $2^{31}$ Bytes of memory with your new cell phone. How much memory is this in gigabytes (GB)?

**Solution**

$$2^{31} = 2 \times 2^{30} = 2 \text{ GB}$$

5. (4 points) Is it possible for a result to overflow when adding a positive and negative number? Explain briefly.

**Solution**

No. The magnitude of the sum will always be less than the largest of the two numbers being added. Note: it was not stated explicitly but the assumption is that the positive or negative number should fit into the N-bit container.

6. (9 points) Add the following 12-bit two’s complement numbers and write the result in hexadecimal. Does the result overflow the 12-bit result.

   (a) $0x03A + 0xCDE$
   (b) $0xF03 + 0x805$
   (c) $0xFFF + 0xFF0$

**Solution**

You must show your work to receive credit, i.e. the binary addition steps.

   (a) (3 points) $0x03A + 0xCDE = 0xD18$; No overflow
   (b) (3 points) $0xF03 + 0x805 = 0x708$; Yes overflow
   (c) (3 points) $0xFFF + 0xFF0 = 0xFE0$; No overflow

7. (9 points) Add the following 12-bit unsigned binary numbers and write the result in hexadecimal. Does the result overflow the 12-bit result.

   (a) $0x03A + 0xCDE$
   (b) $0xF03 + 0x805$
   (c) $0xFFF + 0xFF0$

**Solution**

Note: the binary bit string is the same in this problem as in Problem 6. The difference is the interpretation of the resulting sum.

   (a) (3 points) $0x03A + 0xCDE = 0xD18$; No overflow
   (b) (3 points) $0xF03 + 0x805 = 0x708$; Yes overflow
   (c) (3 points) $0xFFF + 0xFF0 = 0xFE0$; Yes overflow

8. (8 points) Extend the following 4-bit values to 7-bits using sign-extension. Write the final result in hexadecimal.

   (a) $0xA$
   (b) $0x7$
   (c) $0x3$
   (d) $0xF$
Solution

There is a little ambiguity here since hex will need 8-bits but we specify 7-bit extensions. The 8th bit can be ignored in the hex representation. However, it is generally good practice to use appropriate extension to keep the values the same whether viewed as 7 or 8 bits.

(a) (2 points) 0xA = 0x7A or 0xFA

(b) (2 points) 0x7 = 0x07

(c) (2 points) 0x3 = 0x03

(d) (2 points) 0xF = 0x7F = 0xFF
9. (4 points) Consider the previous Problem.
   (a) Are the 4-bit values equal to the corresponding sign-extended 7-bit values when interpreted as two’s complement numbers? Explain briefly.
   (b) Are the 4-bit values equal to the corresponding sign-extended 7-bit values when interpreted as unsigned numbers? Explain briefly.

Solution
   (a) (2 points) Yes, they are equal value with sign-extension in two’s complement numbers. Negative numbers have a 1 in the sign bit.
   (b) (2 points) No, they are not equal with sign-extension in unsigned numbers. If a one value is extended then it makes a much larger unsigned number.

10. (8 points) Extend the following 4-bit values to 7-bits using zero-extension. Write the final result in hexadecimal.
   (a) 0xA
   (b) 0x7
   (c) 0x3
   (d) 0xF

Solution
   (a) (2 points) 0xA = 0x0A
   (b) (2 points) 0x7 = 0x07
   (c) (2 points) 0x3 = 0x03
   (d) (2 points) 0xF = 0x0F

11. (4 points) Consider the previous Problem.
   (a) Are the 4-bit values equal to the corresponding zero-extended 7-bit values when interpreted as two’s complement numbers? Explain briefly.
   (b) Are the 4-bit values equal to the corresponding zero-extended 7-bit values when interpreted as unsigned numbers? Explain briefly.

Solution
   (a) (2 points) No, they are not equal value with sign-extension in two’s complement numbers. A negative number would be turned into a positive number with zero-extension.
   (b) (2 points) Yes, they are equal with sign-extension in unsigned numbers. Zero extended bits do not add to the value of the number.

12. (8 points) Represent the following numbers in two’s complement using a minimum number of bits. Write your final result in binary and hexadecimal.
   (a) 22
   (b) -17
   (c) -5
   (d) -54

Solution
   Note: extra sign-extended bits might be required for a negative number to represent with a hex digit.
(a) (2 points) $22 = 10110 = 0x16$
(b) (2 points) $-17 = 101111 = 0xEF$
(c) (2 points) $-5 = 1011 = 0xB$
(d) (2 points) $-54 = 1001010 = 0xCA$

13. (12 points) Draw the symbol, Boolean equation, and truth table for the following: (You may draw a single truth table with columns for (a)-(d)).

(a) (3 points) 4-input OR gate
(b) (3 points) 4-input NOR gate
(c) (3 points) 4-input XOR gate
(d) (3 points) 4-input XNOR gate

**Solution**

Table 1: Truth Table for Problem 13

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<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>(a) OR</th>
<th>(b) NOR</th>
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![Figure 1: Problem 13](image_url)
14. (4 points) Can a CMOS gate drive a LVCMOS gate reliably? If so, state the low and high noise margins. If not, explain why concisely.

**Solution**

Yes, this will work. The noise margins are

\[
NM_H = V_{OH-CMOS} - V_{IH-LVCMOS} = 3.84 - 1.8 = 2.04V \\
NM_L = V_{IL-LVCMOS} - V_{OL-CMOS} = 0.9 - 0.33 = 0.57V
\]

15. (4 points) Can a TTL gate drive a CMOS gate reliably? If so, state the low and high noise margins. If not, explain why concisely.

**Solution**

No, this will not work. \(V_{OH-TTL} = 2.4\) V, which is lower than \(V_{IH-CMOS} = 3.15\) V which means that a HIGH (i.e., a logical “1”) value can not be received reliably. Note the negative noise margin below \(NM_H\).

The noise margins are

\[
NM_H = V_{OH-TTL} - V_{IH-CMOS} = 2.4 - 3.15 = -0.75V < 0 \\
NM_L = V_{IL-CMOS} - V_{OL-TTL} = 1.35 - 0.4 = 0.95V
\]