On the Construction of Combined k-Fault-Tolerant Hamiltonian Graphs

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A graph G is a combined k-fault-tolerant Hamiltonian graph (also called a combined k-Hamiltonian graph) if $G - F$ is Hamiltonian for every subset $F \subset (V(G) \cup E(G))$ with $|F| = k$. A combined k-Hamiltonian graph G with $|V(G)| = n$ is optimal if it has the minimum number of edges among all n-node k-Hamiltonian graphs. Using the concept of node expansion, we present a powerful construction scheme to construct a larger combined k-Hamiltonian graph from a given smaller graph. Many previous graphs can be constructed by the concept of node expansion. We also show that our construction maintains the optimality property in most cases. The classes of optimal combined k-Hamiltonian graphs that we constructed are shown to have a very good diameter. In particular, those optimal combined 1-Hamiltonian graphs that we constructed have a much smaller diameter than that of those constructed previously by Mukhopadhyaya and Sinha, Harary and Hayes, and Wang et al. © 2001 John Wiley & Sons, Inc.

Keywords: combined k-Hamiltonian graphs; node expansion; diameter

1. INTRODUCTION

Let $G = (V, E)$ be an undirected *graph*, where $V(G)$ is the *node set* and $E(G)$ is the *edge set* of G. The *degree* of a vertex v in G, denoted by $d_G(v)$, is the number of edges adjacent to v. Let $d(u, v)$ denote the distance of vertices u and v. The *diameter* of a graph is the maximum distance among all pairs of nodes. A *path* is a sequence of nodes such that two consecutive nodes are adjacent. A path is represented by $\langle v_0, v_1, v_2, \ldots, v_{t-1}\rangle$. We also write the path $\langle v_0, v_1, v_2, \ldots, v_{t-1}\rangle$ as $\langle v_0 \to P_1 \to v_i, v_{i+1}, \ldots, v_j \to P_2 \to v_k, v_{k+1}, \ldots, v_{t-1}\rangle$, where $P_1 = \langle v_0, v_1, \ldots, v_i\rangle$ and $P_2 = \langle v_j, v_{j+1}, \ldots, v_k\rangle$. A *Hamiltonian path* is a path whose nodes are distinct and span V. A *cycle* is a path of at least three nodes such that the first node is the same as the last node. A cycle is called a *Hamiltonian cycle* if it traverses every node of V exactly once. A graph G is called a *Hamiltonian graph* if it contains a Hamiltonian cycle.

A graph H is said to be a *combined k-fault-tolerant graph of the graph G* (also called *combined k-fault-tolerant G-graph*) if the removal of any combination of p nodes and q edges with $p + q = k$ from H gives rise to a graph which contains a spanning subgraph isomorphic to G. If G is a Hamiltonian cycle, then H is said to be a *combined k-Hamiltonian graph*. A combined k-fault-tolerant graph with n nodes is *optimal* if it contains the least number of edges among all combined k-fault-tolerant graphs with the same number of nodes. The concept of a combined k-fault-tolerant G-graph is a generalization of those of p-node-fault-tolerant G-graphs and of q-edge-fault-tolerant G-graphs which have been studied extensively in VLSI design, parallel architecture, and communication networks design.

Previous results have been focused mostly on the construction of either p-node-fault-tolerant or q-edge-fault-tolerant graphs. Mukhopadhyaya and Sinha [6] and Harary and Hayes [3, 4] constructed families of opti-

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mal combined 1-Hamiltonian graphs. Wang et al. [12] constructed a family of optimal n-node combined 1-Hamiltonian graphs for all even n. Wong and Wong [13] and Paoli et al. [8] proposed a class of graphs $G(n,t)$ and showed that these graphs are optimal t-node-fault-Hamiltonian and optimal t-edge-fault-Hamiltonian for n even and odd, respectively. Although the graph $G(n,t)$ is both optimal t-node-fault-Hamiltonian and optimal t-edge-fault-Hamiltonian, it is not necessary that $G(n,t)$ is an optimally combined t-Hamiltonian. Sung et al [10] showed that $G(n,k)$ is optimally combined k-Hamiltonian for $k = 2,3$ and conjectured that the same is true for all $k \geq 4$. In the construction of an optimal p-node, q-edge, or combined k-Hamiltonian graph, one does not only look for optimality in the number of edges but also strives to find optimal graphs with a smaller diameter. Now, we compare the diameter of the aforementioned optimal combined 1-Hamiltonian graphs. The graphs presented by Mukhopadhyaya and Sinha [6] have diameter $\lfloor \frac{n}{6} \rfloor + 2$ for n even and $\lfloor \frac{n}{8} \rfloor + 3$ for n odd. The graphs proposed by Harary and Hayes [3, 4] and by Wang et al. [12] have diameter $\lfloor \frac{n+1}{4} \rfloor$ and $O(\sqrt{n})$, respectively. The diameter of $G(n,k)$ is $O(n)$ if k is considered as a constant.

In this paper, we initiate a consistent approach and study of the combined k-fault-tolerant G graphs. We concentrate on the construction of combined k-fault-tolerant Hamiltonian graphs. In Section 2, we give some preliminary results. Then, in Section 3, we successfully construct classes of combined k-Hamiltonian graphs from smaller ones by introducing the concept of node expansion. We show in Section 4 that the optimal combined 1-Hamiltonian graphs constructed have a much better diameter than that of those constructed by Mukhopadhyaya and Sinha [6], Harary and Hayes [3, 4], and Wang et al. [12].

2. PRELIMINARY RESULTS

In this section, we will discuss some fault-tolerant properties of complete graphs. These properties are necessary for constructing combined k-Hamiltonian graphs. Let K_n denote a complete graph of n vertices.

Lemma 1. *The graph $K_n - F$ has a Hamiltonian path for $F \subset E(K_n)$ with $|F| \leq n-2$.*

Proof. Let $F' = F - \{e\}$ in which $e \in F$. We only have to show that the graph $K_n - F'$ has a Hamiltonian cycle. Ore's theorem [7] states that a graph G of m nodes has a Hamiltonian cycle if $d_G(x) + d_G(y) \geq m$ for every pair of nonadjacent vertices x, y in G. Let u and v be two nonadjacent vertices of $K_n - F'$. Since $|F'| \leq n-3$ and u, v is adjacent in K_n, the edge (u,v) must be in F'. The maximum number of other edges that can be removed which are incident with u or v is $|F'| - 1$. Thus, $d_{K_n-F'}(u) + d_{K_n-F'}(v) \geq d_{K_n}(u) - 1 + d_{K_n}(v) - 1 - (|F'| - 1) \geq (n-2) + (n-2) - (n-4) = n$. Hence, $K_n - F'$ has a Hamiltonian cycle. Therefore, $K_n - F' - \{e\} = K_n - F$ has a Hamiltonian path, and this lemma follows. □

Theorem 1. *Let $K_n = (V,E)$ be an n-node complete graph and $F \subset (V \cup E)$ be a faulty set with $|F| \leq n-2$. There exists a set $V' \subseteq V(K_n - F)$ with $|V'| = n - |F|$ such that every pair of vertices in V' can be joined by a Hamiltonian path.*

Proof. We prove this theorem by induction on n. This statement can be easily verified for $n = 3$ and 4. Assume that the statement holds for all K_j with $3 \leq j \leq n-1$ and $n \geq 5$.

First, we consider that $|F \cap V(K_n)| = i > 0$. Then, the graph $K_n - F$ is isomorphic to $K_{n-i} - F'$ for some $|F'| \leq |F| - i$. By induction hypotheses, there exists a set $V' \subseteq V$ with $|V'| = n - i - |F'| \geq n - |F|$ such that every pair of vertices in V' can be joined by a Hamiltonian path of $K_{n-i} - F'$. Thus, the statement is also true for the graph $K_n - F$ since $K_{n-i} - F'$ is isomorphic to $K_n - F$.

Next, we consider that $F \subset E$. When $|F| = n-2$, it follows from Lemma 1 that the graph $K_n - F$ has a Hamiltonian path. Thus, there exists a set $V' \subset V$ with $V'| = 2$ such that the pair of vertices in V' can be joined by a Hamiltonian path of $K_n - F$. Now consider that $F \subset E$ and $|F| \leq n-3$. Let H denote the subgraph of K_n given by (V,F). Since $\sum_{v \in V} d_H(v) \leq 2(n-3)$, there exists a vertex $v \in V$ with $d_H(v) \leq 1$. We distinguish the following two cases:

Case 1. There exists a node v with $d_H(v) = 0$.

In other words, all of the edges in $K_n - F$ incident at v are not in F. Thus, the graph $K_n - v - F$ is isomorphic to $K_{n-1} - F$. By induction hypotheses, there exists a subset $V' \subseteq (V - \{v\})$ with $|V'| = n - 1 - |F|$ such that every two distinct nodes $x, y \in V'$ can be joined by a Hamiltonian path of $K_n - v - F$. Let P be a Hamiltonian path of $K_n - v - F$ joining x and y which is written as $\langle x, x' \to P' \to y \rangle$, where x' is a node adjacent to x and P' is a path from x' to y. Then, $\langle x, v, x' \to P' \to y \rangle$ and $\langle v, x, x' \to P' \to y \rangle$ form two Hamiltonian paths of $K_n - F$ from x to y and from v to y, respectively. Since x and y are arbitrary nodes in V', there always exists a Hamiltonian path of $K_n - F$ joining every pair of nodes in $V' \cup \{v\}$. Thus, this statement is true.

Case 2. There exists a node v with $d_H(v) = 1$.

Since there is exactly one edge of $K_n - F$ incident at v which is also in F, it follows that the graph $K_n - v - F$ is isomorphic to the graph $K_{n-1} - F^*$, where $|F^*| = |F| - 1$. By induction hypotheses, there exists a subset $V' \subseteq (V - \{v\})$ with $|V'| = n - |F|$ such that every pair of nodes $x, y \in V'$ can be joined by a Hamiltonian path of $K_n - v - F$. Let P^* be a Hamiltonian path of $K_n - v - F$ joining x and y, which is written as $\langle x = u_0, u_1, \ldots, u_{n-2} = y \rangle$. Since $n \geq 5$, there exists u_j, $0 \leq j \leq n-3$, such that $(v, u_j) \notin F$ and $(v, u_{j+1}) \notin F$.

Then, $\langle x = u_0, u_1, \ldots, u_j, v, u_{j+1}, \ldots, u_{n-2} = y\rangle$ forms a Hamiltonian path of $K_n - F$ joining x and y. Hence, every pair of nodes in V' can be joined by a Hamiltonian path of $K_n - F$.

Thus, the theorem is proved. □

3. NODE EXPANSION

Let $G = (V, E)$ be an undirected graph with node set V and edge set E. Let x be a node in V with degree t. The set $\{x_1, x_2, \ldots, x_t\}$ consists of the neighborhood nodes of x. The *t-node expansion* $X(G, x)$ *of* G *on* x is the graph obtained from G by replacing x by the complete graph K_t, where $V(K_t) = \{k_1, k_2, \cdots, k_t\}$, with the edges $(x, x_i), i = 1, 2, \ldots, t$ deleted from G and the edges $(k_i, x_i), i = 1, 2, \ldots, t$ added to $X(G, x)$. The node expansion is degree preserving, that is, $d_{X(G,x)}(v) = d_G(x) = t$ for all $v \in V(K_t)$ and $d_{X(G,x)}(u) = d_G(u)$ for all $u \in (V - \{x\})$. In particular, if G is d-regular, $X(G, x)$ is also d-regular. The graphs G and $X(G, x)$ are illustrated in Figure 1. Let $N_G(x)$ be the set $\{x\} \cup \{(x, x_i) |$ for all $1 \le i \le t\}$ of G and $M_G(x)$ be the set of $V(K_t) \cup E(K_t) \cup \{(k_i, x_i) | i = 1, 2, \ldots, t\}$ of G.

Lemma 2. *Given* $F_1 \subset (V(G - x) \cup E(G - x))$. *If we delete any* f *edges of* $N_G(x)$ *from the graph* $G - F_1$ *such that the remaining graph is Hamiltonian for* $f \le t - 2$, *then the graph* $X(G, x) - (F_1 \cup F_3)$ *is Hamiltonian, where* F_3 *is a subset of* $M_G(x)$ *and* $|F_3| = f$.

Proof. Since $F_3 \subset M_G(x)$ and $|F_3| \le t - 2$, there exists a set $V' \subset V(K_t)$ of size $t - f'$ such that every two distinct nodes in V' can be joined by a Hamiltonian path of the graph $K_t - F_3$ for $f' = |F_3 \cap (V(K_t) \cup E(K_t))|$. We define a faulty set F_2 of G as follows:

$$F_2 = \{(x, x_i) \mid k_i \notin V' \text{ or } (x_i, k_i) \in F_3, 1 \le i \le t\}.$$

Thus, $|F_2| \le (|V(K_t)| - |V'|) + (|F_3| - f') = t - (t - f') + (f - f') = f \le t - 2$. The graph $(G - F_1) - F_2$ is Hamiltonian since we delete any $t - 2$ edges of $N_G(x)$ from $G - F_1$ such that the remaining graph is Hamiltonian. Thus, there is a Hamiltonian cycle $C = \langle x_i, x, x_j \to P \to x_i\rangle$ in the graph $G - (F_1 \cup F_2)$, where P is a path from x_j to x_i. By the definition of F_2, k_i and k_j are in V' and $(x_i, k_i), (x_j, k_j)$ are not in F_3. Thus, there exists a Hamiltonian path P' joining k_i and k_j in the graph $K_t - F_3$. Therefore, $\langle x_i, k_i \to P' \to k_j, x_j \to P \to x_i\rangle$ forms a Hamiltonian cycle in the graph $X(G, x) - (F_1 \cup F_3)$. This lemma is proved. □

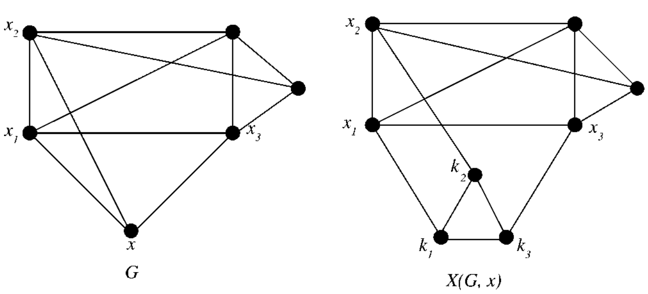

FIG. 1. The graph G and $X(G, x)$.

Theorem 2. *Let* x *be a vertex of* $G = (V, E)$ *with* $d_G(x) = k + 2$. *If* G *is combined k-Hamiltonian, then* $X(G, x)$ *is also combined k-Hamiltonian.*

Proof. Let F be an arbitrary faulty set of the graph $X(G, x)$, where $|F| \le k$. Let $F_1 = F \cap (V(G - x) \cup E(G - x))$ and $F_3 = F - F_1$. Since G is combined k-Hamiltonian, for every $F_2 \subset N_G(x)$ where $|F_2| = |F| - |F_1| = |F_3|$, the graph $G - (F_1 \cup F_2)$ is Hamiltonian. Applying Lemma 2, we can obtain that the graph $X(G, x) - (F_1 \cup F_3)$ is Hamiltonian. Therefore, $X(G, x)$ is combined k-Hamiltonian. The theorem is proved. □

Corollary 1. *Let* $G = (V, E)$ *be a graph of* n *vertices. Let* x *and* y *be two distinct vertices in* G *where* $d_G(x) = d$ *and* $d \le d_G(y) \le d + 1$. *If* G *is an optimal combined* $(d - 2)$*-Hamiltonian and the graph* $G - y$ *is d-regular,* $X(G, x) = (V^*, E^*)$ *is an optimal combined* $(d - 2)$*-Hamiltonian.*

Proof. If G is combined $(d - 2)$-Hamiltonian, it follows from Theorem 2 that $X(G, x)$ is combined $(d - 2)$-Hamiltonian. If $d_G(y) = d$, G is regular since $G - y$ is d-regular. Thus, $X(G, x)$ is a d-regular and optimal combined $(d - 2)$-Hamiltonian. If $d_G(y) = d + 1$, it follows that both d and n are odd. Therefore, $|V^*| = n - 1 + d$ is also odd. Since d-node expansion is degree preserving, $d_{X(G,x)}(y) = d_G(y) = d + 1$ and $d_{X(G,x)}(v) = d$ for all $v \in V^* - \{y\}$. Therefore, $X(G, x)$ is optimal combined $(d - 2)$-Hamiltonian. □

Applying Corollary 1, we can obtain other optimal combined k-Hamiltonian graphs from some known optimal combined k-Hamiltonian graphs by $(k + 2)$-node expansion.

The node expansion of $G = (V, E)$ on the set $U \subset V$, denoted by $X(G, U)$, is a graph that is obtained from G by a sequence node-expansion operations on every node $u \in U$.

Lemma 3. *If the graph* $G = (V, E)$ *is* $(k + 2)$*-regular and k-edge-Hamiltonian, then the graph* $X(G, U) - F$ *is Hamiltonian for every* $F \subset (V(X(G, U)) \cup E(X(G, U)) - V)$ *for* $U \subseteq V$ *and* $|F| \le k$.

Proof. Let v be a node of U and $U' = U - \{v\}$. Assume that the graph $X(G, U') - F'$ is Hamiltonian for every $F' \subset (V(X(G, U')) \cup E(X(G, U')) - V)$ for $|F'| \le k$. Let $F_3 = F \cap M_{X(G,U')}(v)$ and $F_1 = F - F_3$. Therefore, the graph which is deleted any $|F_3|$ edges, denoted by F_2, of $N_{X(G,U')}(v)$ from the graph $G - F_1$ is Hamiltonian since $(F_1 \cup F_2)$ is a subset of $(V(X(G, U')) \cup E(X(G, U')) - V)$ and $|F_1 \cup F_2| = |F| \le k$. Applying Lemma 2, we can obtain that the graph $X(G, U) - (F_1 \cup F_3)$ is Hamiltonian since $F = F_1 \cup F_3$

is an arbitrary subset of $(V(X(G,U)) \cup E(X(G,U)) - V)$ and $|F| \leq k$, this lemma is proved. □

Theorem 3. *If the graph $G = (V,E)$ is a $(k+2)$-regular and optimal k-edge-Hamiltonian, then the graph $X(G,V) = (V^*,E^*)$ is $(k+2)$-regular and optimal combined k-Hamiltonian.*

Proof. Applying Lemma 3, we can obtain that the graph $X(G,V) - F$ is Hamiltonian for every $F \subset (V(X(G,V)) \cup E(X(G,V)) - V)$ for $|F| \leq k$. In fact, $(V(X(G,X)) \cup E(X(G,X))) \cap V = \emptyset$. Thus, $(V(X(G,X)) \cup E(X(G,X))) - V = V(X(G,X)) \cup E(X(G,X))$. Therefore, $X(G,V)$ is combined k-Hamiltonian. Moreover, $X(G,V)$ is an optimal combined k-Hamiltonian since it is $(k+2)$-regular. This theorem is proved. □

It is known that a hypercube, denoted by $Q(n)$, is an n-regular, node symmetric, and link symmetric graph with diameter n. Moreover, $Q(n)$ is shown to be $(n-2)$-edge Hamiltonian graph in [1, 9]. We can obtain Corollary 2 applying Theorem 3.

Corollary 2. *Let $Q(n) = (V,E)$ be an n-dimensional hypercube. The graph $X(Q(n),V)$ is an optimal combined $(n-2)$-Hamiltonian and node symmetric graph with $n \cdot 2^n$ vertices, degree n, and diameter $2n$.*

The star graph, denoted by $S(n)$, is also a famous interconnection network. It is a $(n-1)$-regular, node symmetric, and edge symmetric graph whose vertex number is $n!$ and diameter is $\lfloor 3(n-1)/2 \rfloor$. In [11], the authors show that $S(n)$ is an $(n-3)$-edge Hamiltonian graph. Applying Theorem 3, we also can obtain Corollary 3.

Corollary 3. *Let $S(n) = (V,E)$ be an n-dimensional star graph. The graph $X(S(n),V)$ is an optimal combined $(n-3)$-Hamiltonian and node symmetric graph with $n \cdot n!$ vertices, degree $(n-1)$, and diameter $2\lfloor 3(n-1)/2 \rfloor$.*

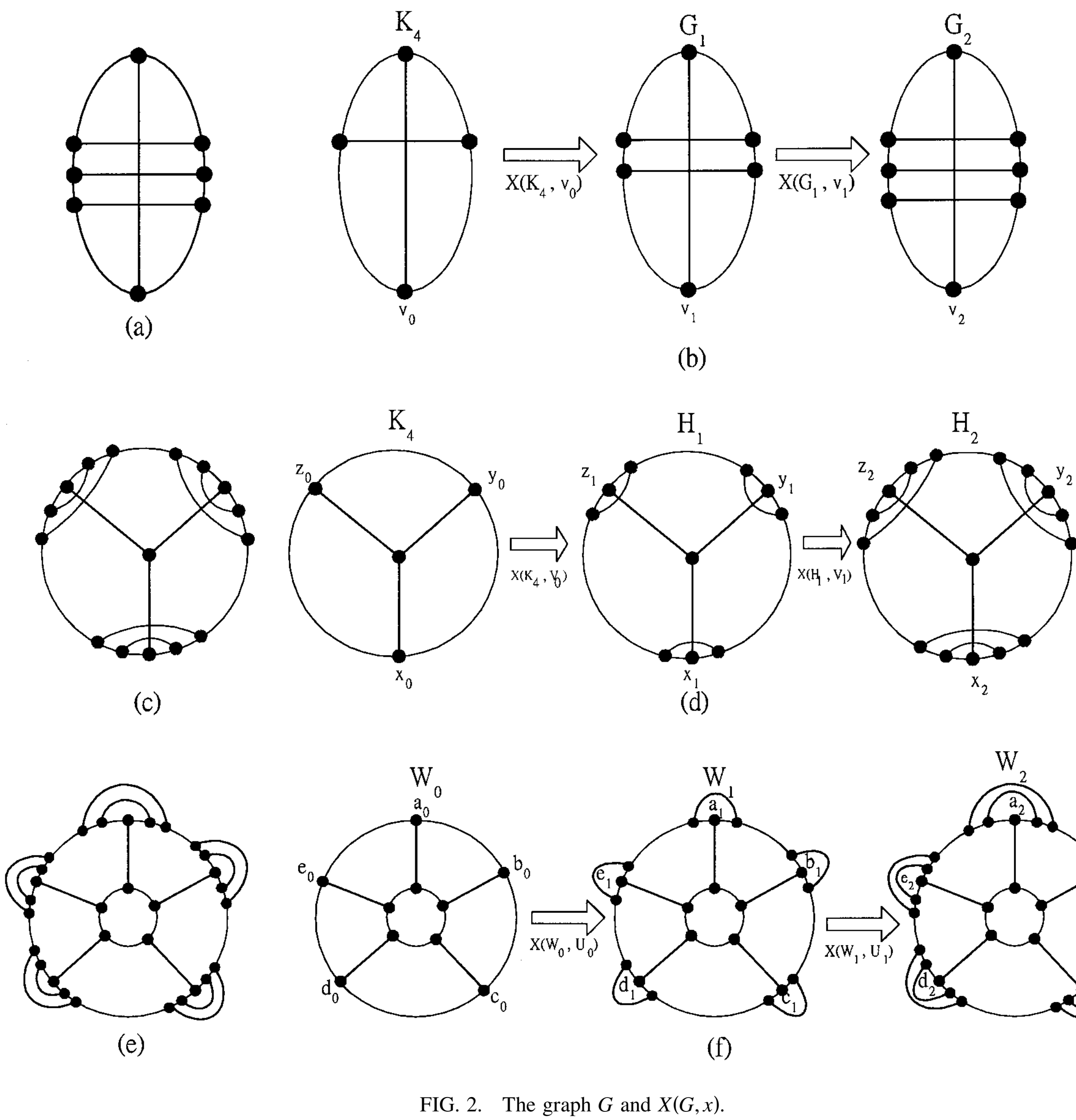

FIG. 2. The graph G and $X(G,x)$.

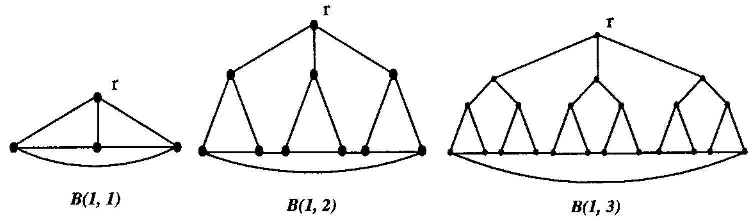

FIG. 3. The graphs $B(1,1), B(1,2)$, and $B(1,3)$

The combined 1-Hamiltonian graphs obtained in [3, 4, 6, 12] can be constructed with the concept of node expansion. In [3, 4], Harary and Hayes presented the optimal combined 1-Hamiltonian graphs H_n [H_8 is illustrated in Fig. 2 (a)]. These graphs can be constructed by a sequence of node expansion on v_i from K_4, as illustrated in Figure 2 (b). Mukhopadhyaya and Sinha [6] proposed the optimal combined 1-Hamiltonian graphs M_n [M_{16} is illustrated in Fig. 2 (c)]. These graphs can be constructed by a sequence of node expansions on $V_i = \{x_i, y_i, z_i\}$ from K_4, as illustrated in Figure 2 (d). The optimal combined 1-Hamiltonian graphs W_n [W_{30} is illustrated in Fig. 2 (e)], proposed in [12], can also be constructed by a sequence of node expansion on $U_i = \{a_i, b_i, c_i, d_i, e_i\}$, as illustrated in Figure 2 (f).

4. DIAMETER

In Section 3, we show that the concept of a node expansion can be applied to construct ,in a fairly special fashion, the optimal combined 1-Hamiltonian graphs obtained independently by Mukhopadhyaya and Sinha [6], Harary and Hayes [3, 4], and Wang et al. [12]. In this section, we will show that classes of optimal combined k-Hamiltonian graphs that we constructed in Section 3 have a very good diameter property. In the special case of $k = 1$, those optimal combined 1-Hamiltonian graphs that we constructed have a much smaller diameter than that of those constructed by Mukhopadhyaya and Sinha [6], Harary and Hayes [3, 4], and Wang et al. [12].

Using Corollary 1, we can easily obtain other optimal combined k-Hamiltonian graphs from an known optimal combined k-Hamiltonian graph by a $(k + 2)$-node expansion on a vertex of degree $k + 2$. Since the complete graph K_{k+3} of $k + 3$ nodes is $(k + 2)$-regular and is the smallest optimal combined k-Hamiltonian graph, the graphs obtained by a sequence of $(k + 2)$-node expansion from K_{k+3} are also $(k + 2)$-regular and, thus, optimal combined k-Hamiltonian. One possible sequence of $(k+2)$-node expansion to construct the optimal combined k-Hamiltonian graphs $B(k, s)$ is as follows:

TABLE 1. The diameter of $B(1,s), M_n, H_n$, and W_n.

	$B(1,s)$	M_n [6]	H_n [3, 4]	W_n [12]
number of vertices	n	n	n	n
diameter	$2\log_2 n - c$	$O(n)$	$O(n)$	$O(\sqrt{n})$
degree	3	3 or 4	3 or 4	3

Procedure **B**(k, s)
  $G = K_{k+3}$
  Pick any vertex r as the root of G
  for $i = 1$ to $s - 1$ do
    $B = \{v \mid d(v, r) = i\}$
    For all $v \in B$
      $G = X(G, v)$

The graphs $B(1,1), B(1,2)$, and $B(1,3)$ are shown in Figure 3. The node labeled with r indicates the root assigned by Procedure **B**(k, s). It can be verified that the number of nodes in $B(k, s)$ is $\frac{(k+2)(k+1)^s-2}{k}$. Moreover, the distance between a node v to the root r is at most s. Thus, the diameter of $B(k, s)$ is at most $2s$. In other words, we have constructed a family of optimal combined k-Hamiltonian graphs with diameter $2\log_{k+1} n - c$.

The diameter for $B(k, s)$ when $k = 1$ is compared to those for Mukhopadhyaya and Sinha [6], Harary and Hayes [3, 4], and Wang et al. [12] in Table 1.

5. CONCLUDING REMARKS

In this paper, we present a general construction scheme, node expansion, for combined k-Hamiltonian graphs for all $k \geq 1$. Using the concept of node expansion, we show that the diameter of the optimal combined 1-Hamiltonian graph $B(1, s)$ is less than those presented in [3, 4, 6, 12]. Furthermore, the graphs presented in [3, 4, 6, 12] can be constructed with node expansion from some smaller graphs.

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